

THINKING WITH MATHEMATICAL MODELS Linear and Inverse Variation

<p>Instructional Time and Investigations</p>	<p>23 days</p>	<ul style="list-style-type: none"> • Inv. 1: Exploring Data Patterns (3 Problems) • Inv. 2: Linear Models and Equations (5 Problems) • Inv. 3: Inverse Variation (4 Problems) • Inv. 4: Variability and Associations in Numerical Data (4 Problems) • Inv. 5: Variability and Associations in Categorical Data (3 Problems)
<p>Goals</p>	<p>Linear and Nonlinear Relationships: Recognize and model linear and nonlinear relationships in bivariate data.</p> <ul style="list-style-type: none"> • A function is a special relationship between values; each input value gives back exactly one output value. A function can be used to create a model of a data pattern. Function models allow you to answer questions or make predictions about a relationship between two variables. Linear relationships are functions. Inverse variation relationships are not linear, but they are functions. 	<p>Data Analysis: Measure variation in data and strength of association in bivariate data.</p> <ul style="list-style-type: none"> • Data about two variables from real-world observations or experiments can be collected and represented in graphs and tables. These representations are useful for analyzing relationships among data, including the variability in the data. • Data may show a pattern or association between the variables. Sometimes you can fit a line to data, find the equation of the line, and measure how well the line fits the data pattern. This is useful for making predictions about data points not observed. • Categorical data must be analyzed in different ways than numerical data including using 2-way tables to analyze relative frequencies.
<p>Common Core Standards</p>	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</p> <p>8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</p> <p>8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</p> <p>Also 8.EE.C.7, 8.EE.C.7b, 8.EE.C.8, 8.EE.C.8a, 8.EE.C.8c, 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.5, 8.SP.A.3, 8.SP.A.4, A-SSE.A.1a, A-CED.A.1, A-CED.A.2, A-CED.A.4, A-REI.A.1, A-REI.B.3, A-REI.C.6, N-Q.A.1, N-Q.A.2, F-IF.A.1, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IF.C.7e, F-IF.C.8, F-IF.C.9, F-BF.A.1, F-BF.A.1a–b, F-BF.A.2, F-LE.A.1, F-LE.A.1b, F-LE.A.2, F-LE.B.5, S-ID.A.1, S-ID.A.2, S-ID.A.3, S-ID.B.5, S-ID.B.6, S-ID.B.6a–c, S-ID.C.7, S-ID.C.9</p>

THINKING WITH MATHEMATICAL MODELS Linear and Inverse Variation

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Linear and Nonlinear Relationships: Recognize and model linear and nonlinear relationships in bivariate data.</p>	<ul style="list-style-type: none"> Recognizing patterns in tables and graphs and describing those patterns using words and equations (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead</i>) Finding slopes of lines and investigating parallel lines (<i>Moving Straight Ahead</i>) Formulating, reading, and interpreting symbolic rules (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead</i>) Solving problems in geometric and algebraic contexts (<i>Covering and Surrounding; Let's Be Rational; Decimal Ops; Variables and Patterns; Shapes and Designs; Comparing and Scaling; Moving Straight Ahead; Filling and Wrapping</i>) Modeling situations with linear equations (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead</i>) Formulating, reading, and interpreting symbolic rules (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead</i>) Recognizing patterns and proportional relationships (<i>Comparing Bits and Pieces; Variables and Patterns; Comparing and Scaling; Moving Straight Ahead</i>) 	<ul style="list-style-type: none"> Recognizing and comparing functions (<i>Growing, Growing, Growing; Say It With Symbols; It's In the System; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Writing equations to represent functions (<i>Growing, Growing, Growing; Say It With Symbols; It's In the System; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Solving geometric and algebraic problems (<i>Looking for Pythagoras; Growing, Growing, Growing; Butterflies, Pinwheels, and Wallpaper; Say It With Symbols; It's In the System; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Finding exact solutions of linear inequalities (<i>It's In the System</i>) Modeling situations with other functions (<i>Growing, Growing, Growing; Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Comparing functions (<i>Growing, Growing, Growing; Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Solving quadratic equations and systems of equations (<i>Say It With Symbols; It's In the System; Frogs, Fleas, and Painted Cubes; Function Junction</i>)
<p>Data Analysis: Measure variation in data and strength of association in bivariate data.</p>	<ul style="list-style-type: none"> Analyzing data using various representations (<i>Data About Us; Variables and Patterns; Samples and Populations; Moving Straight Ahead</i>) Describing shape of the data (<i>Data About Us; Variables and Patterns; Samples and Populations; Moving Straight Ahead</i>) Describing variability (<i>Data About Us; Variables and Patterns; Samples and Populations; Moving Straight Ahead</i>) Exploring different kinds of data (<i>Data About Us</i>) 	<ul style="list-style-type: none"> Summarizing, representing, and interpreting data on a single count or measurement variable (<i>High School</i>) Interpreting linear models (<i>High School</i>) Summarizing, representing, and interpreting data on two categorical and quantitative variables (<i>High School</i>)

LOOKING FOR PYTHAGORAS The Pythagorean Theorem

<p>Instructional Time and Investigations</p>	<p>21 days</p>	<ul style="list-style-type: none"> • Inv. 1: Coordinate Grids (3 Problems) • Inv. 2: Squaring Off (4 Problems) • Inv. 3: The Pythagorean Theorem (4 Problems) • Inv. 4: Using the Pythagorean Theorem: Understanding Real Numbers (4 Problems) • Inv. 5: Using the Pythagorean Theorem: Analyzing Triangles and Circles (3 Problems)
<p>Goals</p>	<p>Pythagorean Theorem: Understand and apply the Pythagorean Theorem.</p> <ul style="list-style-type: none"> • The Pythagorean Theorem relates the areas of the squares on the sides of a right triangle to the area of the square on the hypotenuse. As a result, the Pythagorean Theorem is useful for finding the length of an unknown side of a right triangle given the length of the other two sides, finding the length of a segment joining any two points on a coordinate grid, and for writing the equation of a circle centered at the origin. • The converse of the Pythagorean Theorem can be used to determine whether a triangle is a right triangle. 	<p>Real Numbers: Understand that the set of real numbers consists of rational and irrational numbers.</p> <ul style="list-style-type: none"> • The relationship between a number and its square root is the same as the relationship between the area of a square and the length of its side. The relationship between a number and its cube root is the same as the relationship between the volume of a cube and the length of one of its edges. • The set of real numbers is comprised of the set of rational numbers and the set of irrational numbers. Decimals that neither repeat nor terminate are called irrational numbers. You can locate irrational numbers on a number line, and you can work with them in the same way as with rational numbers.
<p>Common Core Standards</p>	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>8.NS.A.1: Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.</p> <p>8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</p> <p>8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse.</p> <p>8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p> <p>8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system</p> <p>Also 8.NS.A.2, 8.G.A.4, A-CED.A.1, A-CED.A.2, A-REI.D.10, N-Q.A.3</p>

LOOKING FOR PYTHAGORAS The Pythagorean Theorem

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Pythagorean Theorem: Understand and apply the Pythagorean Theorem</p>	<ul style="list-style-type: none"> Measuring lengths (<i>Covering and Surrounding; Shapes and Designs; Stretching and Shrinking</i>) Working with coordinates (<i>Variables and Patterns; Stretching and Shrinking; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models</i>) Measuring areas of polygons and irregular figures (<i>Covering and Surrounding; Stretching and Shrinking</i>) Applying the formula for area of a square (<i>Covering and Surrounding</i>) Formulating, reading, and interpreting symbolic rules (<i>Covering and Surrounding; Variables and Patterns; Shapes and Designs; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models</i>) Working with the triangle inequality (<i>Shapes and Designs</i>) Solving problems in geometric and algebraic contexts (<i>Covering and Surrounding; Shapes and Designs; Moving Straight Ahead; Thinking With Mathematical Models</i>) 	<ul style="list-style-type: none"> Finding midpoints of line segments (<i>Butterflies, Pinwheels, and Wallpaper</i>) Studying transformations and symmetries of plane figures (<i>Butterflies, Pinwheels, and Wallpaper</i>) Looking for patterns in square numbers (<i>Frogs, Fleas, and Painted Cubes; Function Junction; High School</i>) Formulating and using symbolic rules and the syntax for manipulating symbols (<i>Growing, Growing, Growing; Say It With Symbols; It's In the System; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Solving geometric and algebraic problems (<i>Growing, Growing, Growing; Butterflies, Pinwheels, and Wallpaper; Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Exploring trigonometric functions (<i>High School</i>)
<p>Real Numbers: Understand that the set of real numbers consists of rational and irrational numbers.</p>	<ul style="list-style-type: none"> Understanding fractions and decimals (<i>Comparing Bits and Pieces; Let's Be Rational; Decimal Ops</i>) Representing fractions as decimals and decimals as fractions (<i>Comparing Bits and Pieces; Let's Be Rational; Decimal Ops</i>) Finding slopes of lines and investigating parallel lines (<i>Variables and Patterns; Shapes and Designs; Moving Straight Ahead</i>) 	<ul style="list-style-type: none"> Exploring sampling and approximations (<i>High School</i>) Solving quadratic equations (<i>Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction</i>) Investigating symmetry (<i>Butterflies, Pinwheels, and Wallpaper</i>)

GROWING, GROWING, GROWING Exponential Functions

<p>Instructional Time and Investigations</p>	<p>22 $\frac{1}{2}$ days</p>	<ul style="list-style-type: none"> • Inv. 1: Exponential Growth (3 Problems) • Inv. 2: Examining Growth Patterns (3 Problems) • Inv. 3: Growth Factors and Growth Rates (3 Problems) • Inv. 4: Exponential Decay (3 Problems) • Inv. 5: Patterns With Exponents (5 Problems)
<p>Goals</p>	<p>Exponential Functions: Explore problem situations in which two or more variables have an exponential relationship to each other.</p> <ul style="list-style-type: none"> • Situations that can be modeled by an exponential function show a multiplicative pattern in the table of data; the rate of change grows or decays by a constant factor. Tables and graphs can provide more information about an exponential function and help solve problems. 	<p>Equivalence: Develop understanding of equivalent exponential expressions.</p> <ul style="list-style-type: none"> • There is often more than one way to write an equation. The ability to rewrite an equation as an equivalent expression can be helpful when solving problems involving exponential functions and relationships. • There are rules for working with exponential expressions. These properties of exponents are useful in writing equivalent expressions and particularly when working with values written in scientific notation.
<p>Common Core Standards</p>	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.</p> <p>8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</p> <p>8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> <p>8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> <p>Also 8.EE.A.1, 8.EE.A.2, 8.F.A.1, 8.F.A.3, 8.F.B.4, A-SEE.A.1a-b, A-SSE.B.3c, N-RN.A.1, N-RN.A.2, A-CED.A.1, A-CED.A.2, A-REI.D.10, F-IF.B.4, F-IF.B.6, F-IF.C.7e, F-IF.C.8b, F-IF.C.9, F-BF.A.1a, F-LE.A.1a, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-LE.B.5</p>

GROWING, GROWING, GROWING Exponential Functions

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Exponential Functions: Explore problem situations in which two or more variables have an exponential relationship to each other.</p>	<ul style="list-style-type: none"> Looking for graphical or symbolic models to describe a pattern in data (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models</i>) Reasoning relationships such as connections among attributes of geometric figures (<i>Covering and Surrounding; Shapes and Designs</i>) Representing relationships with words, tables, graphs, and equations (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models</i>) Exploring the significance of shapes of graphs and patterns in tables (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models</i>) Attaching meaning to the symbols in a linear equation of the form $y = mx + b$ (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models</i>) Recognizing the significance of constant additive growth (<i>Moving Straight Ahead</i>) Reasoning about percent change (<i>Comparing and Scaling</i>) Recognizing and describing situations that can be modeled by linear relationships (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models</i>) 	<ul style="list-style-type: none"> Extending the analysis to include all positive real numbers for the domain (<i>Function Junction; High School</i>) Using tabular, graphical, and symbolic methods to solve problems that involve exponential functions such as finding half-life or solving equations of the type $ax = b$ (<i>Function Junction; High School</i>) Exploring the significance of shapes of graphs and patterns in tables (<i>Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction</i>); extending the experiences to include recognition of trigonometric relationships (<i>High School</i>) Making sense of the symbols in quadratic relationships, expressed in expanded or factored form (<i>Frogs, Fleas, and Painted Cubes; Function Junction; High School</i>) Reviewing and extending the analysis of exponential and quadratic functions (<i>Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction; High School</i>) Analyzing symbolic expressions of trigonometric and logarithmic functions (<i>Frogs, Fleas, and Painted Cubes; Function Junction; High School</i>) Recognizing the significance of the pattern of change in quadratic relationships (<i>Say It With Symbols; High School</i>); analyzing patterns of change in exponential and trigonometric functions (<i>High School</i>) Recognizing and describing situations that can be modeled by quadratic functions (<i>Say It With Symbols; Frogs, Fleas, and Painted Cubes; Function Junction; High School</i>); extending recognition to trigonometric functions (<i>High School</i>)
<p>Equivalence: Develop understanding of equivalent exponential expressions.</p>	<ul style="list-style-type: none"> Using exponents to express large and small quantities (<i>Prime Time, Looking for Pythagoras</i>) 	<ul style="list-style-type: none"> Applying rules for exponents to interpret more complex algebraic expressions and exponential equations (<i>Function Junction; High School</i>)

FROGS, FLEAS, & PAINTED CUBES Quadratic Functions

Instructional Time and Investigations	$21\frac{1}{2}$ days	<ul style="list-style-type: none"> • Inv. 1: Introduction to Quadratic Functions (3 Problems) • Inv. 2: Quadratic Expressions (4 Problems) • Inv. 3: Quadratic Patterns of Change (4 Problems) • Inv. 4: Frogs Meet Fleas on a Cube: More Applications of Quadratic Functions (4 Problems)
Goals	<p>Quadratic Functions: Explore problem situations in which two variables are in a quadratic relationship.</p> <ul style="list-style-type: none"> • A quadratic function has a unique pattern of change; as the independent variable increases by a constant amount, the second differences of the dependent variable change by a constant amount. The unique characteristics of a quadratic relationship between two quantities are recognizable from a table of ordered pairs, from a graph, and from an equation. You can translate among the various representations to determine which is most useful to solve a problem. • When one variable is dependent on the other, a function can model the data pattern. Functions allow you model real-world situations, answer questions, and make predictions about a relationship. Quadratic relationships are functions. 	<p>Equivalence: Develop understanding of equivalent exponential expressions.</p> <ul style="list-style-type: none"> • The ability to rewrite an equation can be helpful when solving problems involving quadratic relationships and when comparing quadratic equations. You can use the Distributive Property to rewrite quadratic equations in factored form and expanded form; you can then choose the form that will help you determine characteristics of the function or solve a given problem.
Common Core Standards	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.</p> <p>A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it.</p> <p>F-IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.</p> <p>F-IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>F-BF.A.1: Write a function that describes a relationship between two quantities.</p> <p>Also N-Q.A.1, A-SSE.A.1a-b, A-SSE.B.3, A-CED.A.1, A-CED.A.2, A-REI.D.10, F-IF.C.7, F-IF.C.7a, F-IF.C.8, F-IF.C.8a, F-IF.C.9, F-BF.A.1, F-BF.A.1a, F-LE.A.1, F-LE.A.1a-b</p>

FROGS, FLEAS, & PAINTED CUBES Quadratic Functions

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Quadratic Functions: Explore problem situations in which two variables are in a quadratic relationship.</p>	<ul style="list-style-type: none"> Analyzing linear and exponential relationships among quantitative variables (<i>Variables and Patterns; Moving Straight Ahead; Comparing and Scaling; Thinking With Mathematical Models; Growing, Growing, Growing</i>) Comparing patterns of change in tables and graphs for linear and exponential relationships (<i>Moving Straight Ahead; Comparing and Scaling; Thinking With Mathematical Models; Growing, Growing, Growing</i>) Understanding the significance of x- and y-intercepts of a linear function (<i>Moving Straight Ahead; Thinking With Mathematical Models</i>) Understanding the significance of y-intercept in exponential functions (<i>Growing, Growing, Growing</i>) 	<ul style="list-style-type: none"> Reviewing and extending the analysis of quadratic relationships, with more emphasis on symbolic methods (<i>Say It With Symbols; Function Junction; High School</i>) Extending the analysis of patterns of change to other polynomial and trigonometric functions (<i>Function Junction; high school</i>) Understanding the significance of zeroes in solving equations and of maximum and minimum in applications; using symbolic methods for finding zeroes, maximum, and minimum; and applying the quadratic formula (<i>Function Junction; High School</i>)
<p>Equivalence: Develop an understanding of equivalent quadratic expressions.</p>	<ul style="list-style-type: none"> Attaching contextual meaning to m and b in linear relationships $y = mx + b$, and to a and b in exponential relationships $y = a(b)^x$ (<i>Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing</i>) Understanding the significance of scale in constructing and interpreting graphs from data (<i>Data About Us; Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing</i>) 	<ul style="list-style-type: none"> Attaching contextual meaning to different forms of linear and quadratic relationships (<i>Say It With Symbols; It's In the System; Function Junction</i>) Extending symbol sense about quadratics to include meaning and use of the quadratic formula and the formula for the vertex; attaching contextual meaning to different symbolic forms of polynomial and trigonometric functions (<i>Function Junction; High School</i>) Exploring issues of practical and theoretical domain and range, formally treated (<i>Function Junction; High School</i>)

BUTTERFLIES, PINWHEELS, AND WALLPAPER Symmetry and Transformations

<p>Instructional Time and Investigations</p>	<p>18 days</p>	<ul style="list-style-type: none"> • Inv. 1: Symmetry and Transformations (4 Problems) • Inv. 2: Transformations and Congruence (3 Problems) • Inv. 3: Transforming Coordinates (5 Problems) • Inv. 4: Dilations and Similar Figures (4 Problems)
<p>Goals</p>	<p>Transformations: Describe types of transformations that relate points by the motions of reflections, rotations, and translations; and describe methods for identifying and creating symmetric plane figures.</p> <ul style="list-style-type: none"> • Various transformations affect distances and angles of figures differently. These effects help you compare figures and determine the similarity or congruence between figures. 	<p>Congruence and Similarity: Understand congruence and similarity and explore necessary and sufficient conditions for establishing congruent and similar shapes.</p> <ul style="list-style-type: none"> • Two shapes are congruent if a specific sequence of rigid transformations will transform one shape to the other. Two figures are similar if a specific sequence of rigid transformations and dilation will transform one shape to the other. • Properties of transformations, congruence, and similarity can be used to solve problems about shapes and measurements.
<p>Common Core Standards</p>	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations.</p> <p>8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p> <p>8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p> <p>Also: 8.EE.B.6, 8.G.A.1a–c, 8.G.A.5</p>

BUTTERFLIES, PINWHEELS, AND WALLPAPER Symmetry and Transformations

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Transformations: Describe types of transformations that relate points by the motions of reflections, rotations, and translations; and describe methods for identifying and creating symmetric plane figures.</p>	<ul style="list-style-type: none"> • Recognizing and completing mirror reflections (<i>Shapes and Designs</i>) • Recognizing and completing designs with rotation symmetry (<i>Shapes and Designs</i>) • Recognizing, analyzing, and producing tessellations (<i>Shapes and Designs; Stretching and Shrinking</i>) 	<ul style="list-style-type: none"> • Recognizing symmetry in graphs of functions (<i>Say It With Symbols; Function Junction; High School</i>) • Applying the ideas of symmetry to other subjects, such as graphic design and architecture (<i>High School</i>)
<p>Congruence and Similarity: Understand congruence and similarity and explore necessary and sufficient conditions for establishing congruent and similar shapes.</p>	<ul style="list-style-type: none"> • Looking for regularity and using patterns to make predictions (<i>all Connected Mathematics Units</i>) • Relating similarity transformations to the concept of similarity (<i>Stretching and Shrinking</i>) • Performing and analyzing similarity transformations (<i>Stretching and Shrinking</i>) • Describing similarity transformations in words and with coordinate rules (<i>Stretching and Shrinking</i>) • Reasoning about angles formed by parallel lines and transversals (<i>Shapes and Designs</i>) 	<ul style="list-style-type: none"> • Making inferences and predictions based on observation, and proving predictions (<i>High School</i>) • Describing symmetry in graphs, such as graphs of quadratic functions, periodic functions, and power functions (<i>Say It With Symbols; Function Junction; High School</i>) • Reasoning about congruence theorems in geometry (<i>High School</i>) • Finding equations for similar and congruent circles (<i>High School</i>) • Using matrices to represent transformations (<i>High School</i>) • Proving theorems about lines and angles (<i>High School</i>)

SAY IT WITH SYMBOLS Making Sense of Symbols		
Instructional Time and Investigations	21 days	<ul style="list-style-type: none"> • Inv. 1: Making Sense of Symbols: Equivalent Expressions (4 Problems) • Inv. 2: Combining Expressions (4 Problems) • Inv. 3: Solving Equations (4 Problems) • Inv. 4: Looking Back at Functions (4 Problems) • Inv. 5: Reasoning With Symbols (3 Problems)
Goals	<p>Equivalence: Develop understanding of equivalent expressions and equations.</p> <ul style="list-style-type: none"> • Equivalence is useful when solving equations and problems. Equivalent expressions can be generated using properties of operations. Examining equivalent forms of an expression can reveal new information about the context of a problem. • Equivalent expressions can be used to develop and relate formulas for geometric shapes including volumes of cones, spheres, and cylinders. • Algebraic equations and expressions can be used to solve problems. • Equations can have one solution, no solution, or an infinite number of solutions, which can be identified by examining the equation or its graph. 	<p>Functions: Develop understanding of specific functions such as linear, exponential, and quadratic functions.</p> <ul style="list-style-type: none"> • The underlying pattern of change in a relationship or function can be represented symbolically with an equation. Different types of functions, such as linear, inverse, exponential, or quadratic, have specific characteristics in their symbolic representations.
Common Core Standards	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>8.EE.C.7: Solve linear equations in one variable.</p> <p>8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</p> <p>8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> <p>8.G.C.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</p> <p>Also 8.EE.A.2, 8.EE.C.7a–b, 8.EE.C.8, 8.EE.C.8a–c, 8.F.A.1, 8.F.A.2, N-RN.B.3, N-Q.A.1, N-Q.A.2, A-SSE.A.1, A-SSE.A.1a–b, A-SSE.A.2, A-SSE.B.3, A-SSE.B.3a, A-CED.A.1, A-CED.A.2, A-REI.A.1, A-REI.B.3, A-REI.B.4, A-REI.B.4b, A-REI.C.6, A-REI.D.10, A-REI.D.11, F-IF.C.9, F-BF.A.1, F-BF.A.1a, F-LE.A.1, F-LE.A.2, F-LE.B.5</p>

SAY IT WITH SYMBOLS Making Sense of Symbols

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Equivalence: Develop an understanding of equivalent expressions and equations.</p>	<ul style="list-style-type: none"> Using the appropriate order of operations in evaluating expressions and writing symbolic sentences; using parentheses and properties of real numbers to communicate effectively (<i>Prime Time; Variables and Patterns; Accentuate the Negative; Moving Straight Ahead; Filling and Wrapping; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Making sense of linear, quadratic, exponential, and other symbolic expressions (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Evaluating and making sense of symbolic expressions (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Writing and interpreting symbolic sentences (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Reasoning with equivalent expressions (<i>Comparing Bits and Pieces; Let's Be Rational; Variables and Patterns; Shapes and Designs; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Predicting patterns of change (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Solving linear and quadratic equations using tables, graphs, and simple symbolic rules (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Modeling and solving problems (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) 	<ul style="list-style-type: none"> Making sense of linear relationships of the form $ax + by = c$ and linear inequalities (<i>It's In the System; Function Junction</i>) Making sense of polynomial, logarithmic, trigonometric, and rational symbolic expressions and functions (<i>Function Junction; High School</i>) Writing equivalent linear relationships, systems of linear equations and linear inequalities (<i>It's In the System; Function Junction</i>) Writing equivalent expressions involving polynomial, logarithmic, trigonometric, and rational expressions that communicate reasoning using the properties of real numbers (<i>Function Junction; High School</i>) Reasoning with linear relationships and inequalities (<i>It's In the System; Function Junction</i>) Reasoning with equivalent expressions to solve problems that can be modeled by polynomial, logarithmic, trigonometric, and rational functions (<i>Function Junction; High School</i>) Solving linear inequalities and systems of linear equations (<i>It's In the System; Function Junction</i>) Developing a deeper understanding of solving linear and quadratic equations and applying and extending the techniques to solving polynomial and rational equations (<i>Function Junction; High School</i>)
<p>Functions: Develop an understanding of specific functions such as linear, exponential and quadratic functions.</p>	<ul style="list-style-type: none"> Modeling and solving problems (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) 	<ul style="list-style-type: none"> Modeling and solving problems using polynomial functions (<i>Function Junction; High School</i>) Modeling and solving problems using logarithmic and trigonometric functions (<i>; High School</i>)

IT'S IN THE SYSTEM Systems of Linear Equations and Inequalities

<p>Instructional Time and Investigations</p>	<p>17 days</p>	<ul style="list-style-type: none"> • Inv. 1: Linear Equations With Two Variables (3 Problems) • Inv. 2: Solving Linear Systems Symbolically (3 Problems) • Inv. 3: Systems of Functions and Inequalities (3 Problems) • Inv. 4: Systems of Linear Inequalities (4 Problems)
<p>Goals</p>	<p>Linear Equations: Develop understanding of linear equations and systems of linear equations.</p> <ul style="list-style-type: none"> • A system of linear equations can be used to solve problems when two or more equations that represent constraints on the variables in a situation are identified. • The solution to a system of linear equations can be found graphically or algebraically. Analyzing the equations and the situation can help you to determine which strategy is most appropriate to apply. 	<p>Linear Inequalities: Develop understanding of graphic and symbolic methods for solving linear inequalities with one and two variables.</p> <ul style="list-style-type: none"> • The strategies for solving linear equations, linear inequalities, and systems of linear equations can be extended to solving systems of linear inequalities using the properties of inequality.
<p>Common Core Standards</p>	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.</p> <p>8.EE.C.8a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</p> <p>8.EE.C.8b: Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.</p> <p>8.EE.C.8c: Solve real-world and mathematical problems leading to two linear equations in two variables.</p> <p>8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</p> <p>Also A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.B.3, A-REI.B.4, A-REI.B.4b, A-REI.C.5, A-REI.C.6, A-REI.C.7, A-REI.D.10, A-REI.D.12</p>

IT'S IN THE SYSTEM Systems of Linear Equations and Inequalities

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Linear Equations: Develop understanding of linear equations and systems of linear equations.</p>	<ul style="list-style-type: none"> Formulating, reading, and interpreting symbolic rules (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Say It With Symbols</i>) Solving problems in geometric and algebraic contexts (<i>Shapes and Designs; Moving Straight Ahead; Thinking With Mathematical Models; Say It With Symbols; Frogs, Fleas, and Painted Cubes</i>) Solving linear equations (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Say It With Symbols</i>) 	<ul style="list-style-type: none"> Using constraints to interpret a real-world situation in linear and nonlinear contexts (<i>High School</i>) Finding areas of bounded regions in the coordinate plane (<i>High School; College</i>) Solving systems of equations beyond linear equations (e.g., a quadratic and a polynomial); solving multi-dimensional systems of linear equations; using matrices and Cramer's Rule to solve systems of linear equations (<i>High School; College</i>)
<p>Linear Inequalities: Develop understanding of graphic and symbolic methods for solving linear inequalities with one and two variables.</p>	<ul style="list-style-type: none"> Working with the triangle inequality (<i>Shapes and Designs</i>) Solving linear equations (<i>Variables and Patterns; Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Say It With Symbols</i>) 	<ul style="list-style-type: none"> Solving multi-dimensional inequalities (<i>High School; College</i>) Finding minimum and maximum values through linear programming; solving systems of inequalities beyond linear functions (<i>High School</i>)

FUNCTION JUNCTION The Families of Functions		
Instructional Time and Investigations	26 days	<ul style="list-style-type: none"> • Inv. 1: The Families of Functions (5 Problems) • Inv. 2: Arithmetic and Geometric Sequences (2 Problems) • Inv. 3: Transforming Graphs, Equations, and Functions (4 Problems) • Inv. 4: Solving Quadratic Equations Algebraically (4 Problems) • Inv. 5: Polynomial Expressions, Functions, and Equations (4 Problems)
Goals	<p>Functions: Understand equivalence of algebraic expressions and functions.</p> <ul style="list-style-type: none"> • Reasoning strategies for linear and exponential functions can be applied to step functions, absolute value functions, piecewise functions, and polynomial functions. • Arithmetic sequences can be represented with linear functions, and geometric sequences can be represented with exponential functions. • By examining the algebraic form of a function, one can predict the shape and position of a function. 	<p>Equivalence: Understand equivalence of algebraic expressions and functions.</p> <ul style="list-style-type: none"> • Quadratic equations can be solved graphically, algebraically, by completing the square, and by using the Quadratic Formula. Selecting the most efficient method is dependent on how the situation is presented, as well as on whether an exact or approximate answer is needed. A quadratic equation has either 1 real number solution (the vertex lies on the x-axis), 2 real number solutions (the x-intercepts), or 2 complex number solutions (no x-intercepts; solutions are expressed in the form $a + bi$). • The reasoning for addition, subtraction, and multiplication of whole numbers can be applied to the operations used to combine polynomial functions.
Common Core Standards	<p>Common Core Standards for Mathematical Practice</p> <p>MP.1: Make sense of problems and persevere in solving them.</p> <p>MP.2: Reason abstractly and quantitatively.</p> <p>MP.3: Construct viable arguments and critique the reasoning of others.</p> <p>MP.4: Model with mathematics.</p> <p>MP.5: Use appropriate tools strategically.</p> <p>MP.6: Attend to precision.</p> <p>MP.7: Look for and make use of structure.</p> <p>MP.8: Look for and express regularity in repeated reasoning.</p>	<p>Common Core Content Standards</p> <p>A-APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p> <p>A-REI.B.4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</p> <p>F-IF.C.7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>F-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p> <p>F-BF.B.3: Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.</p> <p>Also A-SSE.A.1b, A-SSE.B.3, A-SSE.B.3b, A-REI.B.4, A-REI.B.4a, N-Q.A.1, N-Q.A.2, F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a, F-IF.B.8a, F-IF.C.9, F-BF.A.1a, F-BF.A.1b, F-BF.B.4a, F-LE.A.2</p>

FUNCTION JUNCTION The Families of Functions

Content Connections to Other Units

Goals of the Unit	Prior Work	Future Work
<p>Functions: Understand equivalence of algebraic expressions and functions.</p>	<ul style="list-style-type: none"> Understanding different function types (<i>Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes</i>) Identifying and symbolizing patterns (<i>Variables and Patterns; Comparing and Scaling; Thinking With Mathematical Models; Say It With Symbols</i>), linear Relationships (<i>Moving Straight Ahead</i>), and exponential Relationships (<i>Growing, Growing, Growing</i>) Stretching and transforming geometric figures (<i>Stretching and Shrinking; Comparing and Scaling; Butterflies, Pinwheels, and Wallpaper</i>) Connecting algebraic expressions with graphs (<i>Variables and Patterns; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas and Painted Cubes; It's In the System</i>) Working with quadratic functions (<i>Frogs, Fleas and Painted Cubes; Say It With Symbols</i>) Understanding the number system (<i>Comparing Bits and Pieces, Let's Be Rational; Decimal Ops; Accentuate the Negative; Looking for Pythagoras</i>) 	<ul style="list-style-type: none"> Applying function notation in composition of functions; extending function language and notation to new functions such as trigonometric and logarithmic (<i>High School</i>) Using summation and product notation for finite sums and products (<i>High School</i>); and for infinite sums and products (<i>High School; College</i>); working with the Binomial Theorem (<i>High School; College</i>) Using matrices to transform graphs and functions; transforming polynomial functions and their graphs (<i>High School</i>) Understanding how a transformation on a function affects the corresponding derivative and integral functions (<i>High School; College</i>) Solving systems of quadratic equations and inequalities; modeling real-world data using a quadratic functions; understanding properties of parabolas as a conic section; representing complex numbers and their operations on the complex plane; using complex numbers in polynomial identities and equations (<i>High School</i>)
<p>Equivalence: Understand equivalence of algebraic expressions and functions.</p>	<ul style="list-style-type: none"> Developing symbolic notation (<i>Variables and Patterns; Comparing and Scaling; Thinking with Mathematical Models; Frogs, Fleas and Painted Cubes; Say It with Symbols; It's in the System</i>) 	<ul style="list-style-type: none"> Applying function notation in composition of functions, extending function language and notation to new functions such as trigonometric and logarithmic (<i>High School</i>)