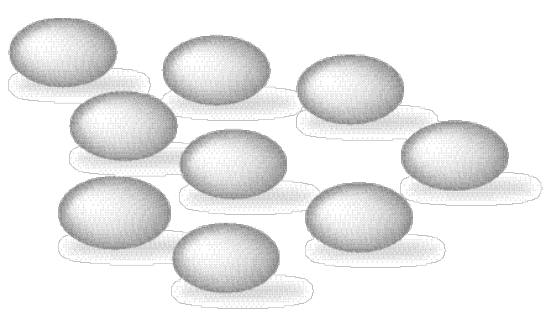
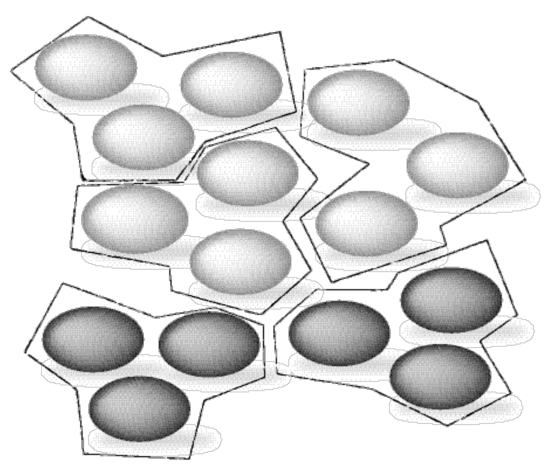
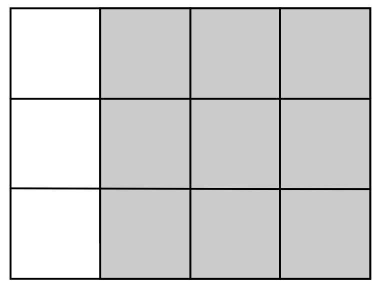
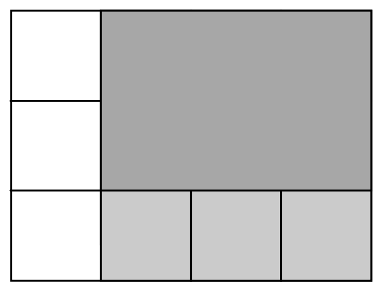
Bits and Pieces II: Homework Examples from ACE
Investigation 1: Questions 12, 23, 33
Investigation 2: Questions 19, 27
Investigation 3: Questions 4, 15, 25
Investigation 4: Questions 8, 11, 20

ACE Question	Possible Answer
ACE Investigation 1	
12. Tell if the sum of the two game cards in the Getting Close Game is closest to 0, 1, 2 or 3. Explain. $\frac{2}{5}$ and $\frac{7}{10}$.	12. $\frac{2}{5}$ is close to $\frac{1}{2}$ but a little less. $\frac{7}{10}$ is between $\frac{1}{2}$ and 1 but closer to $\frac{1}{2}$. So the sum will be closest to $\frac{1}{2} + \frac{1}{2} = 1$.
23. Find 2 fractions with a sum that is between the two given numbers. $1\frac{1}{2}$ and 2.	23. There are many answers for this. Since $\frac{3}{4} + \frac{3}{4} = 1\ \frac{1}{2}$ and $1 + 1 = 2$, and since we want the sum to be between $1\ \frac{1}{2}$ and 2, we might choose two fractions between $\frac{3}{4}$ and 1. For example, $\frac{7}{8} + \frac{9}{10}$.
33. If the beans shown represent $\frac{3}{5}$ of the total beans on the kitchen counter, how many total beans are there on the counter? 	**33**. Since 9 beans represent $\frac{3}{5}$ of the total, each group of 3 beans must be $\frac{1}{5}$ of the total. We need $\frac{2}{5}$ more to make the total. So the total must be 15 beans. 

ACE Investigation 2	
19. Find the value of N that makes each number sentence correct. a. $\frac{2}{3} + \frac{3}{4} = N$ b. $\frac{3}{4} + N = \frac{4}{5}$ c. $N - \frac{3}{5} = \frac{1}{4}$	19. a. Students make a sketch of an area model, or make a number line. (See BP1) Whatever strategy they use they will have to rename the two fractions as twelfths. $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$. b. Students need to be able to rephrase this sentence as $\frac{4}{5} - \frac{3}{4} = N$. Renaming the fractions as twentieths we have $N = \frac{4}{5} - \frac{3}{4} = \frac{16}{20} - \frac{15}{20} = \frac{1}{20}$ c. Again, rephrasing this sentence as $N = \frac{1}{4} + \frac{3}{5} = \frac{5}{20} + \frac{12}{20} = \frac{17}{20}$
27. Tony works at a pizza shop. He cuts 2 pizzas into eight equal sections each. Customers then eat $\frac{7}{8}$ of each pizza. Tony says that $\frac{7}{8} + \frac{7}{8} = \frac{14}{16}$, so $\frac{14}{16}$ of all the pizza was eaten. Is Tony's addition correct? Explain.	27. This is tricky to explain because Tony's conclusion ("$\frac{14}{16}$ of all pizza as eaten") is correct. However, we know that $\frac{7}{8} + \frac{7}{8} = \frac{14}{8}$ not $\frac{14}{16}$. So, where does the contradiction come from? Well, Tony did not say what his unit "whole" was when he wrote his sentence. If we say that $\frac{7}{8}$ of one pizza + $\frac{7}{8}$ of one pizza = $\frac{14}{16}$ of one pizza this is ***not*** correct. If we say that $\frac{7}{8}$ of the total + $\frac{7}{8}$ of the total = $\frac{14}{16}$ of the total this is ***not*** correct. Tony used 2 different "wholes" in his sentence. He CAN CORRECTLY say that $\frac{7}{8}$ of one pizza + $\frac{7}{8}$ of one pizza = $\frac{14}{16}$ of all the pizza. But he needs to say what he is taking a fraction of.
ACE Investigation 3	
4. a. Use a brownie pan model to show whether finding $\frac{2}{3}$ of $\frac{3}{4}$ of a pan of brownies means the same thing as finding $\frac{3}{4}$ of $\frac{2}{3}$ of a pan of	4. a. The "brownie pan model" is actually an area model and refers to the problems students investigated in class.

brownies. b. If the brownie pans are the same size, how do the final amounts of brownies compare in the situations in part (a)? c. What does this say about $\frac{2}{3}$ x $\frac{3}{4}$ and $\frac{3}{4}$ x $\frac{2}{3}$?	First identify $\frac{3}{4}$ of a rectangular area (by drawing vertical lines) then divide this into thirds by drawing horizontal lines. The heavily shaded area is $\frac{2}{3}$ of $\frac{3}{4}$. You can see this is $\frac{6}{12}$. (Students could also notice that there are 3 vertical strips representing $\frac{3}{4}$ of the pan, so $\frac{2}{3}$ would be 2 of these vertical strips, or $\frac{2}{4}$.)   If the $\frac{2}{3}$ is identified first and then lines are drawn dividing this into 4 equal parts, we get the same resulting area.

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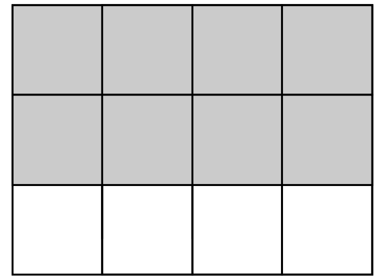
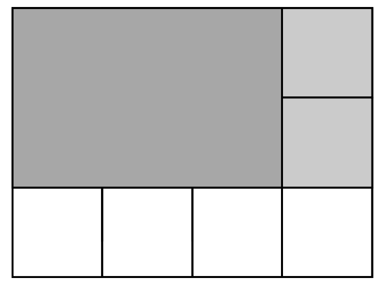
<td>

b. Depending on how students draw this, the first calculation will result in $\frac{6}{12}$ or $\frac{1}{2}$. The second calculation also results in $\frac{6}{12}$.
This model leads to the common shortcut of multiplying the numerators and denominators.
c. Students should be able to connect this idea about multiplying fractions to what they know about multiplying whole numbers: that multiplication is commutative. The reason it is perhaps not immediately obvious is the use of the word “of” in fraction multiplication.</td>
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<td>15. Find each product. Look for patterns to help you.
a. $\frac{1}{3}$ x 18

b. $\frac{2}{3}$ x 18

c. $\frac{5}{3}$ x 18

d. $1\frac{2}{3}$ x 18</td>
<td>15.
a. Students might draw 18 objects and group these into three equal-sized groups of 6. So, $\frac{1}{3}$ of 18 = 6.
b. If $\frac{1}{3}$ of 18 = 6, then $\frac{2}{3}$ of 18 should be twice as much, or 12.
c. If $\frac{1}{3}$ of 18 = 6, then $\frac{5}{3}$ of 18 = 5 times as much = 30.
d. If students think of $1\frac{2}{3}$ as the same as $\frac{5}{3}$ they will get the same answer as in part c. Or they may think of this as 1 x 18 + $\frac{2}{3}$ x 18 and get 18 + 12 = 30.</td>
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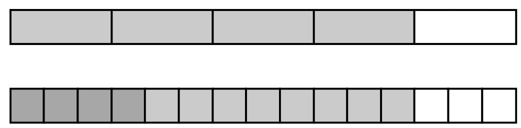
25. Use your algorithm for multiplying fractions to find the product. $10\frac{3}{4} \times 2\frac{2}{3}$	25. Students might do this by renaming as $\frac{43}{4} \times \frac{8}{3}$ and then multiplying the numerators and denominators, to get $\frac{344}{12} = 28\frac{8}{12}$. OR They might think of $\frac{1}{4}$ of $\frac{8}{3} = \frac{2}{3}$ and so $\frac{43}{4}$ must be 43 times as much, or $43(\frac{2}{3}) = \frac{86}{3} = 28\frac{2}{3}$ (as before). OR They might think of this as $10(2\ \frac{2}{3}) + \frac{3}{4}\ (2\ \frac{2}{3})$ $= 10(2\ \frac{2}{3}) + (3 \text{ times } \frac{1}{4} \text{ of } \frac{8}{3})$ $= (20 + \frac{20}{3}) + 3(\frac{2}{3})$ $= 20 + \frac{20}{3} + \frac{6}{3}$ $= 22 + 6\ \frac{2}{3}$ $= 28\ \frac{2}{3}$ Students have various logical ways to think of this multiplication.
30. Choose the number that, when multiplied by $\frac{4}{7}$, will be less than $\frac{4}{7}$. Choose from: a. $\frac{1}{7}$ b. $\frac{7}{7}$ c. $\frac{17}{7}$ d. $\frac{8}{7}$	**30.** For this students have to understand that to get a result less than $\frac{4}{7}$ we must be multiplying by anything less than 1. Thus, $\frac{9}{10} \times \frac{4}{7} = \frac{36}{70}$, for example. From the choices available, multiplying $\frac{4}{7}$ by $\frac{1}{7}$ will produce an answer less than $\frac{4}{7}$.
ACE Investigation 4	
8. Divide. Draw a picture to prove that each quotient makes sense. a. $\frac{4}{5} \div 3$ b. $1\frac{2}{3} \div 5$ c. $\frac{5}{3} \div 5$	8. There are two different ways to think of this division. The drawing that students will choose to do depends on how they think about the problem. We might think that this says, "We have $\frac{4}{5}$ of something and we want to divide it up into three pieces. How big is each piece?" This is the partition way of thinking of dividing. OR we might think of this as, "We have $\frac{4}{5}$ of something and we want to find out how many times 3 will go into that." This is the grouping way of

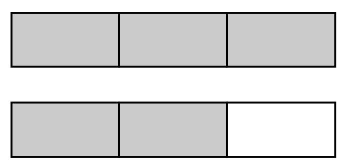
thinking of division. Obviously 3 is too big to make even one group of 3, so the question is, “What part of 3 will go into $\frac{4}{5}$?”

a.

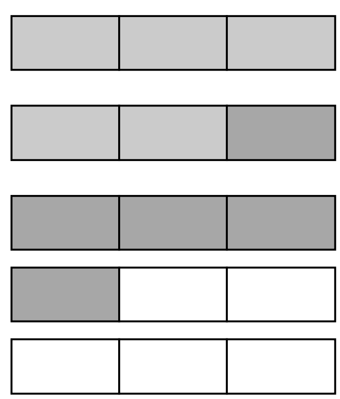
Thinking of this as a partition, we need to find a way to break $\frac{4}{5}$ into 3 groups. One way to do this would be to divide each $\frac{1}{5}$ into 3 pieces. These would be fifteenths, and there would be $\frac{12}{15}$ shaded in the model below. Then we can share these fifteenths into three groups; each group has a size of $\frac{4}{15}$.



b. Thinking of this as $\frac{5}{3}$ to be divided into 5 pieces, and asking, “How big is each piece?” we can see each piece is $\frac{1}{3}$.



If we think of this as, “How many times will 5 fit into $1\frac{2}{3}$?” we have to say that if the group size is 5 then only $\frac{1}{3}$ of the group will fit into $1\frac{2}{3}$.



	c. As above.
11. A latte is the most popular coffee drink at Antonio's Coffee Shop. Antonio makes only one size, and he uses $\frac{1}{3}$ cup of milk to make each drink. For parts (a) to (c) find: • How many lattes he can make with the amount of milk given. • What the remainder means, if there is one. a. $\frac{7}{9}$ of a cup b. $\frac{5}{6}$ of a cup c. 3 $\frac{2}{3}$ cups.	11. a. This asks how many times $\frac{1}{3}$ goes into $\frac{7}{9}$, or $\frac{7}{9} \div \frac{1}{3}$. One way to do this is to rename as $\frac{7}{9} \div \frac{3}{9}$. We know that $\frac{7}{9} \div \frac{1}{9}$ would be 7, so $\frac{7}{9} \div \frac{3}{9}$ would be a third as much, or $\frac{7}{3}$, which is 2 $\frac{1}{3}$ lattes. This says that Antonio can make 2 lattes, using $\frac{2}{3}$ or $\frac{6}{9}$ of a cup of milk. This leaves $\frac{1}{9}$ *of a cup* leftover, which is enough left over to make $\frac{1}{3}$ *of a latte*. b. This asks for $\frac{5}{6} \div \frac{1}{3}$. Reasoning somewhat differently from part a, a student might think $\frac{1}{3}$ of a cup + $\frac{1}{3}$ of a cup = $\frac{2}{3}$ = $\frac{4}{6}$ of a cup of milk. This leaves $\frac{1}{6}$ *of a cup of milk leftover*, or enough for *half a latte.* c. 3 $\frac{2}{3} \div \frac{1}{3} = \frac{11}{3} \div \frac{1}{3} = 11$. This is enough for 11 lattes with no milk leftover.
20. Find the quotient. $2\frac{1}{2} \div 1\frac{1}{3}$	20. Students may do this by renaming with common denominators: $2\frac{1}{2} \div 1\frac{1}{3} = \frac{5}{2} \div \frac{4}{3}$ $= \frac{15}{6} \div \frac{8}{6} = \frac{15}{8}$. (Students might break the last step into two, reasoning that $\frac{15}{6} \div \frac{1}{6} =$ 15, so $\frac{15}{6} \div \frac{8}{6}$ will be an eighth of this or $\frac{15}{8}$.) *This reasoning leads to the common shortcut of multiplying by the denominator and dividing by the numerator of the second fraction.)*