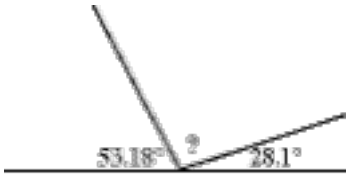
Homework Examples from ACE: *Bits and Pieces III*
Investigation 1: Questions 119, 32, 34, 45
Investigation 2: Questions 9, 17, 28, 45
Investigation 3: Questions 6, 16, 26, 31
Investigation 4: Questions 6, 11, 28

Example from ACE	Possible Solution
Investigation 1	
19. Use fraction benchmarks to estimate the result of each operation: **a.** 2.43 + 1.892 **b.** 4.694 – 1.23 **c.** 12.92 + 3.506 – 6.18	19. a. 2.43 is close to 2.5 which is $2\frac{1}{2}$. 1.892 is close to 2. $2.43 + 1.892 \approx 2\frac{1}{2} + 2 = 4\frac{1}{2} = 4.5$. b. 4.694 is close to 4.75 which is $4\frac{3}{4}$. 1.23 is close to 1.25 which is $1\frac{1}{4}$. $4.694 - 1.23 \approx 4\frac{3}{4} - 1\frac{1}{4}$ $= 3\frac{1}{2} = 3.5$. c. 12.92 is close to 13. 3.506 is close to 3.5 which is $3\frac{1}{2}$. 6.18 is close to 6.25 which is $6\frac{1}{4}$. $12.92 + 3.506 - 6.18 \approx 13 + 3\frac{1}{2} - 6\frac{1}{4} = 10\frac{1}{4} = 10.25$.
32. Find the value of N that makes the mathematical sentence correct. Use fact families to help you. **a.** 2.3 + N = 3.42 **b.** N – 11.6 = 3.75	32. In the context of addition and subtraction, "fact families" record 3 ways that we can see how the same two parts relate to the whole: part A + part B= whole, whole – part A = part B, whole – part B = part A. a. The relationship 2.3 + N = 3.42 can also be written as 3.42 – N = 2.3 and 3.42 – 2.3 = N. The last equation is helpful in finding a value for N. N = 3.42 – 2.3 = 1.12. b. The relationship N – 11.6 = 3.75 can also be written as N – 3.75 = 11.6 and N = 11.6 + 3.75. This last equation is helpful in finding N = 15.35.
34. Find where to place the decimal points in 102 and 19 so that the sum of the two numbers is 1.21.	34. Since the final answer is 1.21, which is close to 1, we know that we can not use 1.9 or 19 or 10.2 as part of the desired sum. 1.02 + 0.19 = 1.21.
45. Find the measures of the	45. Because the given angle is a straight angle we know the

angles marked with question marks.  	sum of the three angles marked must be 180 degrees. So, 53.18 + 28.1 + N = 180. 81.28 + N = 180. N = 180 – 81.28 (using a fact family) N = 98.72 The missing angle is 98.72 degrees.
Investigation 2	
9. First estimate the product. Then, compute the exact result of the product, using fraction multiplication. 3.822×5.2	9. 3.822 x 5.2 ≈ 4 x 5 = 20. Using fraction representations, 3.822 x 5.2 = $\frac{3822}{1000}$ x $\frac{52}{10}$ = $\frac{3822 \times 52}{10000}$ = $\frac{198744}{10000}$ = 19.8744
17. Find the value of N. $3.2 \times N = 0.96$	17. If we only think about the digits in the problem we have 32 x N = 96. This would give N = 3. Now we have to think about place value. 3.2 x 3 = 9.6 3.2 x 0.3 = 0.96. (Thinking about place value means thinking about the meaning of the digits after the decimal point, in terms of fractions with powers of 10 as denominators. 3.2 means $3\frac{2}{10}$ or $\frac{32}{10}$. 0.96 means $\frac{96}{100}$. $\frac{32}{10} \times N = \frac{96}{100}$ so N = $\frac{3}{10}$ or 0.3
28. Use the number sentence $145 \times 326 = 47270$ to help you solve the following problems: **a.** 1.45×32.6 = ____ **b.** 0.326×1450 = ____ **c.** ____ $\times$ 32.6 = 472.7	28. a. $1.45 \times 32.6 = \frac{145}{100} \times \frac{326}{10}$ = $\frac{145 \times 326}{1000}$ = $\frac{47270}{1000}$ = 47.270. (Check: 1.45 is about $1\frac{1}{2}$, and 32.6 is about 30, and we know that $1\frac{1}{2}$ x 30 = 45. So this answer is sensible.)

d. $0.0145 \times$ ____ = 47.27	b. $0.326 \times 1450 = \frac{326}{1000} \times 1450$ $= \frac{326 \times 1450}{1000}$ $= \frac{47270}{1000}$ = 472.700. c. $N \times \frac{326}{10} = \frac{4727}{10}$ $N = \frac{4727}{10} \div \frac{326}{10}$ $N = 4727 \div 326$ $N = 14.5$ Check: $14.5 \times 32.6 = \frac{145}{10} \times \frac{326}{10} = \frac{47270}{100} = 472.7$ d. $0.0145 \times N = 47.27$ $\frac{145}{10000} \times N = \frac{4727}{100}$ $\frac{145}{10000} \times N = \frac{472700}{10000}$ (rewriting to make the denominators all the same) $N = \frac{472700}{10000} \div \frac{145}{10000}$ So, $N = 472700 \div 145 = 3260$ check: $0.0145 \times 3260 = \frac{145}{10000} \times 3260 = \frac{145 \times 3260}{10000} = \frac{472700}{10000} = 47.27$
45. **a**. Find a length and a width for a rectangle with an area of 56 square feet. **b**. Find a length and a width for a rectangle with an area of 5.6 square feet. **c**. Find a length and a width for a rectangle with an area of 0.56 square feet.	45. a. There are several possible dimensions. 1 x 56 = 56. 2 x 28 = 56. 4 x 14 = 56. 7 x 8 = 56. b. Suppose our choice was 7 x 8 for part a. Then we can change this to 0.7 x 8 or 7 x 0.8. c. 0.7 x 0.8.

Investigation 3	
6. For a–d, will the quotient be greater than or less than 1? Explain. **a.** 19.36 ÷ 3.893 **b**. 0.962 ÷ 0.3 **c.** 5.3 ÷ 11.07 **d**. 0.072 ÷ 0.09	6. a. There are several ways to think about this. We might round off and say the answer is approximately 19 ÷ 4 which is more than 1. b. We might think of these as fractions with a common denominator. $\frac{962}{1000} \div \frac{300}{1000}$ is the same as 962 ÷ 300, which is greater than 1. c. We might compare values and say that 5.3 < 11.07, so the quotient will be less than 1.
16. Compute: 45.13 ÷ 0.125	16. $45.13 \div 0.125 = \frac{4513}{100} \div \frac{125}{1000}$ $= \frac{45130}{1000} \div \frac{125}{1000}$ $= 45130 \div 125$ $= 361\frac{5}{125}$ Or we can set up a division problem $125\overline{)45130.00}$ with quotient 361.04
26. **a**. Find the decimal equivalent for each fraction or mixed number: $\frac{11}{9}$ $1\frac{6}{27}$ **b**. Describe the relationship between the fraction, mixed number, and their decimal equivalents.	26. a. Students have learned (in *Bits and Pieces* II) that fractions can be thought of as divisions. Thus $\frac{a}{b}$ can be thought of as a ÷ b. $\frac{11}{9}$ can be thought of as $9\overline{)11.00}$ quotient $1.22\ldots$ 9 20 18 20 b. The values are all the same. If we plotted them on a number line they would all occupy the same position. The fraction (or rational number) is written in the format $\frac{a}{b}$ without regard to whether a > b or a < b. The mixed number follows the convention that if a> b then we should rewrite this to show how many times the denominator divides into the numerator. Thus, if we are dividing by 9, for example, then every group of 9 ninths will give a whole number. Any remainder becomes a leftover number of ninths. The decimal also relies on dividing the denominator into the numerator, but instead of reporting a

	remainder, the division continues into the decimal places until the decimal either terminates or repeats.
31. a. Find the product of 0.37 and 10. b. How is the product similar to 0.37? How is it different? c. Multiply the product from part a by 10. How is the product similar to 0.37? d. In general, what do you think happens to a decimal number when you multiply by 10? e.	31. a. $0.37 \times 10 = \frac{37}{100} \times 10 = \frac{370}{100}$ $= 3\frac{70}{100} = 3.70$. b. The digits are the same but the position of the digits relative to the decimal point has changed (which could also be described as the decimal point moving…but that would not be quite so helpful). Each digit has moved one place left and has a value 10 times higher, which makes sense of the operation. c. $3.7 \times 10 = \frac{37}{10} \times 10 = \frac{370}{10} = 37$. Again the digits remain unchanged, but the new positions of the digits again show that each digit has increased its value, this time by a factor of 100, compared to the original 0.37. d. When you multiply a decimal number by 10 the same digits appear in the result, but the value of each digit has increased by a factor of 10, so each digit has apparently moved one decimal place left.
Investigation 4	
6. Jen and Sarah go to lunch at the Green Grill. Their meal totals $28.00. **a**. If tax is 6%, what will be the total with tax? **b**. They want to leave a 20% tip based on the total before tax. How much tip should they leave? **c**. Describe two strategies that Marie and Sarah can use to figure the amount of the tip.	6. You can think of percents as fractions, or decimals or as rates. Different ways of thinking lead to different strategies. a. We might think of 6% as a rate of 6 cents on each dollar. Scaling this up to $28 we have a tax of 28 x $0.06 = $1.68. The total bill will be $29.68. b. We might think of 20% as a fraction. 20% is $\frac{20}{100}$ or $\frac{1}{5}$. Therefore we need $\frac{1}{5} \times 28 = \frac{28}{5} = 5.60$. The tip is $5.60. c. Instead of thinking of 20% as $\frac{1}{5}$ they might think of this tip in two stages. 10% is $\frac{1}{10}$, and we know that multiplying $28 by $\frac{1}{10}$ will keep the same digits but give them place values that as $\frac{1}{10}$ of the original. The tip would be $2.80 if we used 10% to calculate it. Doubling this we get $5.60 as before.
11. The local boutique is having a sale on hats. Lisa finds a straw hat that is already marked down. The price tag	11. The focus is on the amount of the discount, $9.00. If we think of percent discount as a rate then we need to compare 9 out of 36 to x out of 100. We can use

marked down. The price tag shows that the original price is \$36.00 and the sale price is \$27.00. What percentage has the straw hat been marked down? Explain.	equivalent fractions to solve this: $\frac{9}{36} = \frac{x}{100}$ $\frac{1}{4} = \frac{x}{100}$ $x = 25.$
28. The Maces are shopping in Chicago, Illinois where the state sales tax is $6\frac{1}{4}\%$. If the Maces purchase a \$5.00 map of the city at the tourist center, what is the total cost for the map?	28. There are several ways to think about this problem. • You can think of this as 5% + 1% + $\frac{1}{4}\%$. We know 10% of \$5.00 is \$0.50. So 5% of \$5.00 is \$0.25. We also know that 1% is $\frac{1}{100}$ so 1% of \$5.00 is \$0.05. This leaves $\frac{1}{4}\%$ to think about. If 1% of \$5.00 is \$0.05 then $\frac{1}{4}\%$ of \$5.00 must be about \$0.01. The sales tax will be \$0.25 + \$0.05 + \$0.01 = \$0.31. You then add the tax to the price to get a total of \$5.31, • Or you can think of this as 6.25%. $\frac{6.25}{100} = 0.0625$. 6.25% of \$5.00 = 0.0625 x 5 = \$0.3125. The total price = \$5.31. • Or you can think of the total as 106.25% of the price, and compute 1.0625 x 5 = 5.3125. The total price = \$5.31.