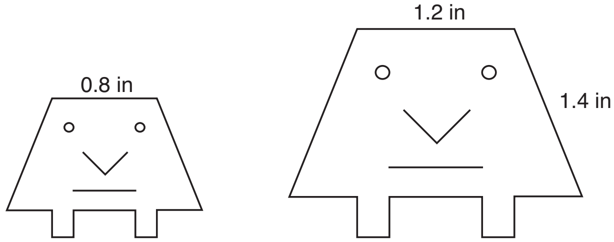
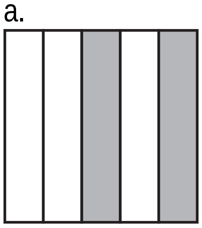
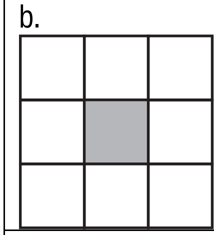
Comparing and Scaling: Homework Examples from ACE
ACE Investigation 1: #5, 13
ACE Investigation 2: #4, 14.
ACE Investigation 3: #8, 12.
ACE Investigation 4: #5, 22.

ACE Question	Possible Answer
ACE Investigation 1	
5. Carlos' Data: Sleeping:18 hours Eating: 2.5 hours Recreation: 8 hours Talking on phone: 2 hours Watching TV: 6 hours Doing chores or homework: 2 hours Other 9.5 hours Decide if each statement is an accurate description of how Carlos spent his time that weekend. a. He spent one sixth of his time watching TV. b. The ratio of hours spent watching TV to hours spent doing chores is 3 to 1. c. Recreation, talking on phone, and watching TV took about 33% of his time. d. Time spent doing chores or homework was only 20% of the time spent watching TV. e. Sleeping, eating and "other" activities took up 12 hours more than all other activities combined.	5. a. The "whole" in this case is "his time," so first we have to calculate the total number of hours involved. This is 48 hours. Since TV time: total time = 6:48 or 1:8 this is not an accurate statement. b. TV time: chores time = 6: 2 = 3:1. This is an accurate statement. c. Recreation + phone time + TV time = 16 hours. His total time = 48 hours. 16:48 = 1:3, or $\frac{1}{3}$ or about 33%. This is an accurate statement. d. Chores time: TV time = 2: 6 = 1:3 or about 33%. This is not an accurate statement. e. Sleeping + eating + "other" = 30 hours. This leaves (48 – 30) hours left. 30 hours is 12 hours more than 18 hours. This is an accurate statement. Note: The way of comparing in part e involves a subtraction to find a difference. Ratios involve dividing to find a factor or rate.
13. Write statements comparing the lengths of corresponding segments in the two Grump drawings. Use each concept at least once.  a. ratio b. fraction c. percent	13. a. The top of the little Grump: top of the large Grump = 0.8:1.2 or, scaling this up, 8: 12 or 2:3. (This is the only comparison possible, since the question specifies *corresponding* parts.) b. The lengths in the smaller drawing are $\frac{2}{3}$ of the lengths in the larger drawing (or the lengths in the larger drawing are $\frac{3}{2}$ or 1.5 times the lengths in the smaller drawing.) c. The lengths in the smaller drawing are 66.7% of the lengths in the larger drawing (or the

d. scale factor.	lengths in the larger drawing are 150% of the lengths in the smaller drawing). d. The scale factor from the small to the large drawing is $\frac{3}{2}$ or 1.5, which means that if you multiply lengths in the small drawing by 1.5 you get the lengths in the larger drawing. (Or we could say that the scale factor from the large to the small drawing is $\frac{2}{3}$, which means that multiplying lengths in the large drawing by $\frac{2}{3}$ will produce the lengths in the small drawing.) Note: When the order in the English sentence about scale factor mentions the small drawing first, and uses the word "to," students may start to write the length on the small drawing first, and come up with the wrong scale factor. It may help to use an arrow to indicate the *action* of the scale factor. Length on small drawing: length on large drawing = 0.8:12 = 2:3. BUT Small drawing _ large drawing, indicates we want to know what factor will *change* 0.8 $\rightarrow$ 1.2 or 2 $\rightarrow$ 3. The arrow represents the *action of multiplying* by the scale factor 1.5.
ACE Investigation 2	
4 . At camp Miriam uses a pottery wheel to make three bowls in 2 hours. Duane makes five bowls in 3 hours. a. Who makes bowls faster, Miriam or Duane? b. At the same pace, how long will it take Miriam to make a set of 12 bowls? c. At the same pace, how long will it take Duane to make a set of 12 bowls?	4. a. One way to make a comparison is to scale these rates up so that either the number of bowls or the time is the same for both Miriam and Duane. Miriam makes 3 bowls in 2 hours, or 6 bowls in 4 hours, or 9 bowls in 6 hours, or 12 bowls in 8 hours, or 15 bowls in 10 hours etc. Duane makes 5 bowls in 3 hours, or 10 bowls in 6 hours, 15 bowls in 9 hours. This means that Miriam takes longer to make 15 bowls than Duane. Duane is faster. (If we make the times equal, then Miriam

	makes 9 bowls in 6 **hours**, while Duane makes 10 bowls in 6 **hours.** So Duane is faster.) b. Miriam makes 12 bowls in 8 hours. c. Scaling up Duane's ratio 5 bowls: 3 hours by multiplying by *whole numbers* does not produce 12 bowls: whole number of hours. We have to think about 5:3 = 12:x. Students might do this by representing the ratios as fractions. $\frac{5}{3} = \frac{12}{x}$, and then renaming the fractions so that both numerators are 60. $\frac{5}{3}$ becomes $\frac{60}{36}$. $\frac{12}{x}$ becomes $\frac{60}{5x}$. So $5x = 36$, so Duane takes $\frac{36}{5}$ or $7\frac{1}{5}$ hours to make 12 bowls. (Students might also solve this by changing 3 hours into 180 minutes and then making a unit rate for Duane of 1 bowl every $\frac{180}{5}$ minutes, that is 36 minutes.)
14. For each shaded diagram, write three statements comparing the shaded and unshaded regions. In one statement, use fraction ideas to express the comparison. In the second, use percent ideas. In the third, use ratio ideas. a.  b. 	14. The question asks for a comparison of shaded: unshaded, not shaded: total. a. The shaded area is $\frac{2}{3}$ the size of the unshaded area; the shaded area is 66.7% of the unshaded area; the shaded area: unshaded area = 2:3. b. The shaded area is $\frac{1}{8}$ of the unshaded area. The shaded area is 12.5% of the area of the unshaded part. Shaded area: unshaded area = 1:8.
ACE Investigation 3	
8. Choose the fastest walker. a. Montel walks 3 miles in 1 hour. b. Jerry walks 6 miles in 2 hours.	8. One way to make this comparison is to make all the distances the same by scaling up each

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c. Phil walks 6 miles in 1.5 hours.
d. Rosie walks 9 miles in 2 hours.
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rate to some multiple of 3, 6 and 9. Say we choose to scale up each ratio to find the time for 18 miles. Then Montel walks 18 miles in 6 hours, Jerry walks 18 miles in 6 hours, Phil walks 18 miles in 4.5 hours, and Rosie walks 18 miles in 4 hours. From this we see that Rosie's rate is fastest. (We could also scale up each ratio to make the times the same, say 6 hours.)

Another way is to make a unit rate for each walker. Montel's rate is 3 miles per 1 hour, Jerry's rate is 3 miles per one hour, Phil's rate is 4 miles per one hour, and Rosie's is 4.5 miles per 1 hour. To find Phil's unit rate a rate table may help. The table below applies only to Phil's rate.
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<tr><td>Dist</td><td>6</td><td>2 (6 divided by 3)</td><td>4 (2 multiplied by 2)</td><td></td></tr>
<tr><td>time</td><td>1.5</td><td>0.5 (1.5 divided by 3)</td><td>1 (0.5 multiplied by 2)</td><td></td></tr>
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OR
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<tr><td>Dist</td><td>6</td><td>12 (6 multiplied by 2)</td><td>4</td><td></td></tr>
<tr><td>Time</td><td>1.5</td><td>3 (1.5 multiplied by 2)</td><td>1</td><td></td></tr>
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12.
Study the data in these rate situations. Then write the key relationship in three ways:

In fraction form with a label for each part
As two different unit rates with a label for each part.

a. Latanya's 15 mile commute to work each day takes an average of 40 minutes.
b. In a 5-minute test, one computer printer produced 90 pages of output.
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12.
a.

$\frac{15\text{ miles}}{40\text{ minutes}}$.
$\frac{\frac{15}{40}\text{ mile}}{1\text{ minute}}$ or $\frac{1\text{ mile}}{\frac{40}{15}\text{ minutes}}$

b.

$\frac{90\text{ pages}}{5\text{ minutes}}$

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c. An advertisement for a Caribbean cruise trip promises 168 hours of fun for only $1344. d. A long-distance telephone call lasts 20 minutes and costs $4.50.	• $\frac{18\text{ pages}}{1\text{ minute}}$ or $\frac{1\text{ page}}{\frac{5}{90}\text{ minute}}$ c. • $\frac{168\text{ hours}}{\$1344}$ • $\frac{\frac{168}{1344}}{\$1}$ or $\frac{1\text{ hour}}{\frac{1344}{168}\text{ dollars}}$ d. • $\frac{20\text{ minutes}}{\$4.50}$ • $\frac{\frac{20}{4.5}\text{minutes}}{\$1}$ or $\frac{1\text{ minute}}{\frac{4.5}{20}\text{dollars}}$ Note: the fractions in the above rates can be rewritten in simpler, equivalent forms. They are left in this form to show the calculation needed.
ACE Investigation 4	
5. Denzel makes 10 of his first 15 shots in a basketball free-throw contest. His success rate stays about the same for his next 100 free throws. Write and solve proportions to answer each part. Round to the nearest whole number. Start each part with the original 10 of 15 free throws. a. About how many free throws does Denzel make in his next 60 attempts? b. About how many does he make in his next 80 attempts? c. About how many attempts does Denzel take to make 30 free throws? d. About how many attempts does he take to make 45 free throws?	5. a. 10 shots made: 15 attempts = ? shots made: 60 attempts. (This can be done by scaling up the ratio, using a factor of 4.) Denzel makes 40 shots. b. Since 15 is not a factor of 80, we need another strategy to solve $\frac{10}{15} = \frac{x}{80}$. We could rename both fractions with equal denominators: $\frac{160}{240} = \frac{3x}{240}$, so 3$x$ = 160, so x = $\frac{160}{3}$ or about 53 free throws. OR, we could write $\frac{10}{15} = \frac{x}{80}$ as $\frac{2}{3} = \frac{x}{80}$ or 0.667 = x ÷ 80. Therefore, x = 80(0.667) or about 53 as before. c. $\frac{10}{15} = \frac{30}{x}$. A scale factor of 3 gives us $\frac{30}{45} = \frac{30}{x}$, so x = 45. d. $\frac{10}{15} = \frac{45}{x}$. We could use a scale factor of 4.5 to rewrite the first fraction as

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$\frac{45}{67.5} = \frac{45}{x}$, so x = 67.5.

OR, we could use a unit rate strategy:
$\frac{10}{15}$ is the same as $\frac{1}{1.5}$.
So $\frac{1}{1.5} = \frac{45}{x}$.
Scaling up the unit rate by a factor of 45 we have $\frac{45}{67.5} = \frac{45}{x}$, so Denzel will have to make 68 attempts to make 45 free throws.

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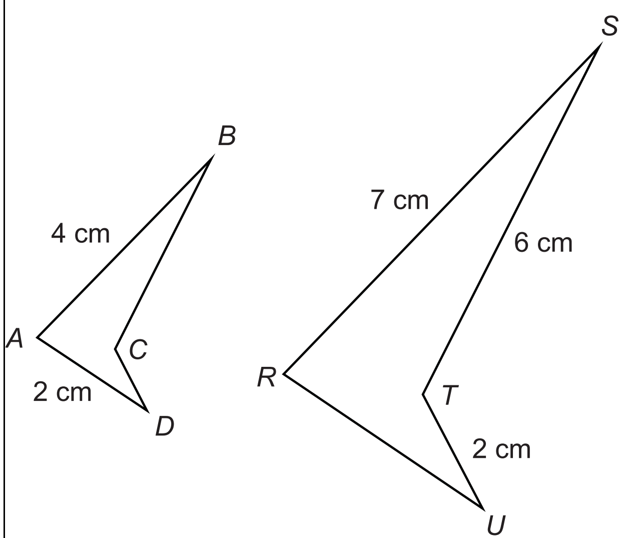
22.
The sketch shows 2 similar polygons.



a. What is the length of the side BC?
b. What is the length of the side RU?
c. What is the length of the side CD?

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22.
This problem is a particular case of solving proportions. Students might use scale factor, or renaming fractions, or some other strategy.

a. The scale factor that transforms the smaller figure to the larger is $\frac{7}{4}$ or 1.75. The scale factor that transforms the larger to the smaller is $\frac{4}{7}$. To find BC we need to shrink the corresponding side ST by a factor of $\frac{4}{7}$.

BC = $(\frac{4}{7})$(ST) = $(\frac{4}{7})(6) = \frac{24}{7} = 3\frac{3}{7}$ cm.

b. To find RU we can apply the scale factor 1.75 to the corresponding side AD, to get
RU = 1.75(2) = 3.5 cm.

OR we can set up and solve $\frac{4}{7} = \frac{2}{x}$ by some other method. We might rename the fractions to have the same numerator:
$\frac{4}{7} = \frac{4}{2x}$. So 2x = 7, so x = 3.5 cm.

c. Comparing corresponding sides,
$\frac{4}{7} = \frac{CD}{2}$.
Renaming with common denominators we have $\frac{8}{14} = \frac{7CD}{14}$. So
7CD = 8, so CD = $\frac{8}{7}$ cm.

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