Homework Examples from ACE: ***How Likely Is It?***

ACE Question	Possible Solution
Investigation 1	
3. Kalvin flipped a coin five days in a row and got tails every time. He told his mother there must be something wrong with the coin. Do you think there is something wrong with the coin? How could Kalvin find out?	3. This question addresses the idea of probability as "what is to be expected over the long term." Kalvin should toss the coin many more times. It is unusual to get 5 tails in a row, but not impossible. If he tossed the coin 100 times and got *many* more tails than heads he might suspect that the coin is not fairly balanced. Theoretically, each toss of a fair coin should have a 50% chance of turning out to be a tail, but we should not be surprised if this 50% figure does not occur over a small number of tosses. (If he repeated the experiment (5 tosses of a fair coin) a hundred times and recorded how many times he got 5 tails in a row he would find that this will occur purely by chance about 3 times in a 100.)
4. Len flipped a coin three times and got heads every time. What are the chances he will get tails on his next toss? Explain your reasoning.	4. The probability of HHHT is the same as the probability of HHHH. Each coin toss is independent of the last toss, even though it seems that some combinations are less likely than others. In other words, the coin has no memory of what the last toss was, and so there is no change in the probability of the outcome of a single toss; each toss has a 50% chance of being H, and a 50% chance of being a T. Note: if we had asked *before* any tosses had taken place whether it was more likely to get 4 heads in 4 tosses, or 3 heads and a tail, then we could say that HHHH was less likely than 3 heads and a tail. But this is because there are 4 ways to get 1 tail: HHHT, HHTH, HTHH, THHH.
9. Kalvin's sister Kyla came up with yet another way for Kalvin to pick his breakfast. She put 1 blue marble and 1 red marble in each of two bags. She explained that each morning Kalvin should choose one marble from each bag. If the marbles are the same color, Kalvin gets to eat Cocoa Blast. If they are different colors, he must eat Health Nut	9. In the first bag there are two equally likely outcomes: red or blue. Likewise for the second bag. Therefore, this situation is exactly like tossing a coin twice or tossing two coins; each bag is analogous to a coin toss, and "red" is analogous to "head" and "blue" to "tail." Note: This question foreshadows the idea of simulation. In simulations a model is chosen

Flakes. Explain how drawing one marble from each of the two bags and tossing two coins are similar.	which has the same underlying probabilities as the situation to be investigated. The purpose in choosing the model is to set up repetitions of an experiment, using the model rather than the real situation, because the model is more convenient.
32. While Yolanda was at a carnival, she watched a game in which a paper cup was tossed. It costs $1 to play the game. If the cup lands upright, the player wins $5. Yolanda watched the cup being tossed 50 times. The cup landed on its side 32 times, upside down 13 times, and upright 5 times. **a.** If Yolanda plays the game 10 times, about how many times can she expect to win? How many times can she expect to lose? **b.** Would you expect Yolanda to have more or less money at the end of 10 games than she had before? Why?	32. a. Yolanda only wins if the cup lands upright. From the experimental data we see that the probability of winning is 5 out of 50, or 10%. Therefore, if Yolanda plays 10 times she can expect to win 10% of 10 times = 1 time. She will lose 9 times. (Note: Ten trials is a very small number of trials, so we should not be surprised if Yolanda's results are very different from the percentages produced by the longer experiment.) b. If Yolanda wins 1 time and plays 10 times, she will have spent $10 to play and won back $5.
Investigation 2	
5. A bag contains several marbles. Some are red, some are white, and some are blue. Carlos counted the marbles and found that the theoretical probability of drawing a red marble is $\frac{1}{5}$ and the theoretical probability of drawing a white marble is $\frac{3}{10}$. **a.** What is the smallest number of marbles that could be in the bag? **b.** Could the bag contain 60 marbles? If so, how many of each color must it contain? **c.** If the bag contains 4 red marbles and 6 white marbles, how many blue marbles must it contain?	5. a. The ratio of red marbles: total number of marbles must be 1:5 since the probability of choosing a red is 1:5. The actual number of red could be 1 in a total of 5, or 2 in a total of 10, or 3 in a total of 15 etc. Likewise the actual number of white could be 3 in a total of 10, or 6 in a total of 20, or 9 in a total of 30. The first ratios that use the same total number of marbles are 2 red in 10 and 3 red in 10. 10 is the lowest total (or the first common denominator). b. Red: total = 1:5 = 12:60. White:total = 3:10 = 18:60. It is possible to make correct ratios with a total of 60 marbles. c. $\frac{\text{Red}}{\text{Total}} = \frac{1}{5}$ or $\frac{4}{?}$ We need to rename the fraction $\frac{1}{5}$ so that the numerator is 4. $\frac{1}{5} = \frac{4}{20}$. Using a total of 20 marbles we have $\frac{\text{White}}{\text{Total}} = \frac{3}{10} = \frac{6}{20}$. So there are 4 red and 6

<table>
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<td>d. How can you determine the probability of drawing a blue marble?</td>
<td>white marbles, leaving 10 blue marbles to complete the total of 20.
d. There are only 3 choices so P(Red) + P(white) + P(blue) = 1.
So $\frac{1}{5} + \frac{3}{10}$ + P(blue) = 1.
So P(blue) = $1 - (\frac{1}{5} + \frac{3}{10}) = 1 - (\frac{5}{10}) = \frac{5}{10}$.</td>
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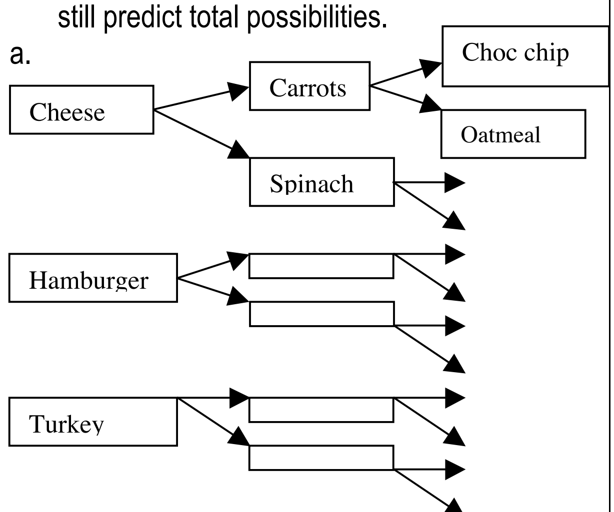
<td>8. Lunch at Casimer consists of one sandwich, one vegetable, and one cookie. The cook has an equal number of each sandwich, vegetable, and cookie. She is not paying any attention to how she puts the lunches together so the students don't know what lunch they will get today. Sage's favorite lunch is a grilled cheese sandwich, carrots, and a chocolate chip cookie.

Create a counting tree to determine how many different lunches are possible. List all the possible outcomes.

b. What is the probability that Sage will get his favorite lunch? Explain your reasoning.

c. What is the probability that Sage will get at least one of his favorite things? Explain your reasoning.</td>
<td>8. It is important to see the logic behind the counting tree, so that we can tell ahead of time how many possible outcomes there will be. If the tree gets large and unwieldy we can still predict total possibilities.
a.


Reading from the right we have 2 choices of cookie, for each of 2 choices of vegetable, for each of 3 choices of sandwich. The list of outcomes is CCC, CCO, CSC, CSO, HCC. HCO, HSC, HSO, TCC, TCO, TSC, TSO.
b. Only one of these, CCC, is Sage's favorite lunch so she has a 1in 12 chance of getting her favorite.
c. CCC, CCO, CSC, CSO, HCC, HCO, HSC, TCC, TCO, TSC all contain at least one of Sage's favorite items. $\frac{10}{12}$.
Note: it would have been easier to enumerate the times when Sage got NONE of her favorites, HSO, TSO.</td>
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<td>11. Pietro and Isabella are playing a game involving tossing a coin three times. Isabella scores 1 point if no two consecutive toss results match (as in HTH). Pietro scores a</td>
<td>11. One way to analyze this game is to list all outcomes with accompanying winner.
<table>
<tr><th>Outcome</th><th>Winner</th></tr>
<tr><td>HHH</td><td>No winner</td></tr>
<tr><td>HHT</td><td>Pietro</td></tr>
<tr><td>HTH</td><td>Isabella</td></tr>
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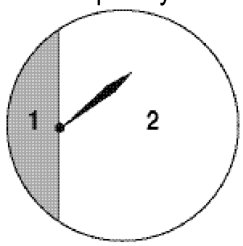
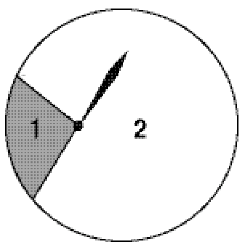
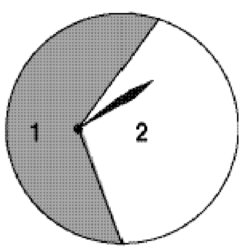
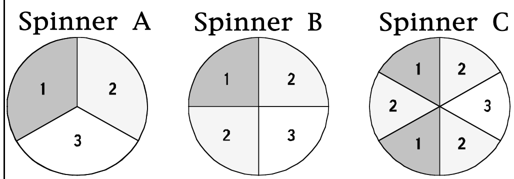
point if exactly two consecutive toss results match (as in HHT). The first player to 10 points wins. Is this a fair game? Explain. If it is not a fair game, change the rules to make it fair.	HTH — Isabella THH — Pietro HTT — Pietro TTH — Pietro THT — Isabella TTT — No winner There are 4 ways that Pietro can win and only 2 ways that Isabella can win. This is not fair. We could change the rules so that Pietro wins on exactly 2 consecutive matches and Isabella wins otherwise, in which case there are 4 ways for Isabella to win also. Or we keep the original rules but award Isabella double points for a win.
35. Suppose you are a contestant on the *Gee Whiz Everyone Wins!* game show in Problem 2.3, and you have already won a mountain bike, a CD player, a vacation to Hawaii, and a one-year supply of Super Soft toilet paper. You have just played the bonus round and lost, but the host makes the following offer: you can draw from the two bags again, but this time you do not need to predict the color. If the two colors match, you will win $5,000. If the two colors do not match, you must return all the prizes you have won. Would you accept this offer? Explain why or why not.	35. There are 9 possible outcomes from choosing 2 blocks: RR, RY, RB, YR, YY, YB, BR, BY, BB. There are 3 ways to win: RR, YY, BB. And 6 ways to lose. P(match) = $\frac{3}{9}$, and P(no match) = $\frac{6}{9}$. It depends on the value of the prizes thus far whether a contestant would risk everything on a 1 in 3 chance of winning \$5000. Suppose the prizes thus far are worth \$1000, then *accepting* the offer will give you $\frac{1}{3}$ chance of winning \$5000 + \$1000, and $\frac{2}{3}$ chance of losing \$1000. If we think of playing this variation of the game 3 times, and accepting the offer, we would expect to win \$6000 once and lose \$1000 twice, an average gain of \$1333 per game. By *rejecting* the offer we would win \$1000 three times, an average gain of \$1000 per game. So, in this case by accepting the offer we can expect to win more than by rejecting the offer. However, if the value of the prizes thus far was higher than \$1000 then our average expected win may be less by accepting than by rejecting. Note: this analysis relies on "average expected win." We know that if we only play the game once then the long term average may not be what we get on one trial. So we might decide to play safe and take what we are sure of.
Investigation 3.	
3. When you spin each of the spinners below, are	3. a. It looks like there is a greater possibility of

the two possible outcomes—landing on a space with 1, landing on a space with 2—equally likely? If not, which outcome has a greater theoretical probability, landing on 1 or landing on 2? Explain your reasoning. **a.**  **b.**  c. 	landing on "2" because the area is larger. But the rotation of the spinner is what creates all the possibilities, and so the possibilities are determined by the *angle of rotation*. Because of the placement of the center of the spinner, in turning 180 degrees clockwise from pointing vertically up to pointing vertically down the spinner sweeps through "2." This is half of a complete rotation, so the outcomes are equally likely. b. Now the amount of rotation needed to sweep through "2" is larger than 180 degrees, so there is a greater chance of "2" than of "1." c. The amount of rotation needed to sweep through "1" is greater than 180 degrees, so "1" has a greater probability that "2."
5. Mollie is designing a game for a class project. She made the three spinners shown here and experimented with them to see which one she liked best for her game. She spun each spinner 20 times and wrote down her results, but she forgot to record which spinner gave which set of data. Which spinner most likely gave each data set? Refer to the data sets on the next page. Explain your answer.	5. Spinner A has 3 equally likely outcomes. We should look for a list that reflects this, knowing that with 20 trials these theoretical probabilities will not occur. The second data set has 7 "1's" and 5"2's" and 8 "3's." This is close to the theoretically expected outcome for spinner A. Spinner B should have "2" occurring half of the time, and "1" and "3" occurring equally often. The third data set has 11 "2's" and 4 "1's" and 5 "3's."



First data set

1 2 3 2 1 1 2 1 2 2 2 3 2 1 2 2 2 3 2 2

Second data set

2 3 1 1 3 3 3 1 1 2 3 2 2 2 1 1 1 3 3 3

Third data set

1 2 3 3 1 2 2 2 3 2 1 2 2 2 3 2 2 3 2 1

Spinner C should produce “2” half the time in the long term, and should produce fewer “3’s” than “1’s. The first data set has 12 “2’s” and 5 “1’s” and 3 “3’s.”

Investigation 4.

6. If Katrina cannot curl her tongue, is it possible that both of her parents can curl their tongues? Why or why not?

6. If Katrina cannot roll her tongue then she has inherited tt from her parents. She inherited t from her mother, so her mother must have had either tt or tT or Tt, but she cannot have had TT. Likewise with her father. If both parents have tT then there is a 1 in 4 chance that their offspring can have tt. Thus, both Katrina’s parents could have tT and be able to roll their tongues.

		Mother	
		t	T
Father	t	tt (Katrina)	tT
	T	Tt	TT

19. Suppose you are trying to determine Dawn and Tomas's earlobe alleles. Here is the information you have:

- Dawn has attached earlobes.
- Tomas has nonattached earlobes.
- Their two daughters have nonattached earlobes.
- Their son has attached earlobes.

a. What are Dawn's earlobe alleles?

b. What are Tomas's earlobe alleles?

c. If they have another child, what is the probability that he or she will have attached earlobes?

19.

a. Dawn has ee.

b. Tomas has EE or Ee, because he needs an "E" to have non-attached earlobes. However, Tomas' son has attached earlobes, so he must have inherited a "e" from EACH parent. Tomas must have at least one "e." So Tomas has Ee.

c. Each time they have a child the same probabilities come to bear on what the child will inherit. This should be a 50% probability for Dawn and Tomas.

		Dawn	
		e	e
Tomas	E	Ee	Ee
	e	ee	ee

The existence of 2 girls who already have Ee does not make it more or less likely that another child will have this trait. In other words, there is no natural force at play in EACH birth trying to keep things in balance. In the LONG run we will have 50% of the offspring with ee if their parents are both Ee.