Vocabulary: Prime Time

Concept

Factor: A whole number that divides into another whole number evenly (that is, with no remainder and a whole number quotient).

Factor Pair: A pair of factors that can be multiplied to make a target number.

Multiple: A whole number that can be divided by a given whole number factor.

Prime Number: A prime number has factors of only 1 and itself. (1 and 0 are not primes.) **Composite Number**: A number with factors other than 1 and itself.

Prime Factorization: A prime factorization of a number is the list of prime numbers that would multiply together to make the given number.

Fundamental Theorem of Arithmetic: This theorem says that the prime factorization of any number is unique.

Example

4 is a *factor* of 24 because $4 \times ? = 24$ leads to the conclusion that the other factor is the whole number 6. We can use the term *divisor* interchangeably with *factor*.

2 and 12 are a *factor pair* for 24, as are 3 and 8.

24 is a *multiple* of 8, because 8 is a factor of 24.

19 is a *prime* because the only factors are 1 and 19, but 9 is not a prime because it has factors 1, 3, 9.

14 is a *composite* number because 14 has factors 2 and 7, as well as 1 and 14.

The prime factorization of 24 is $2 \times 2 \times 2 \times 3$, where all of the factors are prime. It is true that $24 = 3 \times 8$ or $2 \times 4 \times 3$ or $6 \times 2 \times 2$ etc. But, these last three factorizations are not prime factorizations, because "8" and "4" and "6" are not prime numbers. See below for an example of finding a prime factorization of 40.

If we start with 40 and think of this as 2 x 20, then we should "break down" 20 further, since it is not prime. Thus, $40 = 2 \times 20 = 2 \times 2 \times 10 = 2 \times 2 \times 2 \times 5$. This is a prime factorization of 40. If we started with a different factor pair, say $40 = 5 \times 8$, and then "broke down" 8 further, we would get $40 = 5 \times 8 = 5 \times 2 \times 2 \times 2$. The resulting prime factorization of 40 is the same (except for order) no matter how we begin on the process.

Greatest Common Factor: A *common factor* is any number that is a factor that 2 or more whole numbers. One way to find common factors is to make prime factorizations of the two numbers in question.

Lowest Common Multiple: The LCM is the lowest whole number that is a multiple of 2 or more whole numbers. One way to find the LCM is to make prime factorizations of the target numbers.

Relatively Prime: Two numbers are *relatively prime* if they have no prime factors in common other than 1.

Rectangular Array: One way of modeling multiplication of factor pairs is to picture these as the dimensions of a rectangle.

 $36 = 2 \times 2 \times 3 \times 3$ and $84 = 2 \times 2 \times 3 \times 7$. We see that the factors 2, 2 and 3 appear in both prime factorizations; so 2, 3, 6 (combining 2 and 3), 4 (combining 2 and 2) and 12 (combining 2 and 2 and 3) are common factors of both 36 and 84. 12 is the greatest of these so 12 is the GCF.

 $36 = 2 \times 2 \times 3 \times 3$ and $84 = 2 \times 2 \times 3 \times 7$. We need the lowest combination of prime factors that will include ALL of the prime factors for each of these numbers. If we start with $2 \times 2 \times 3 \times 3$ (to "cover" 36) then we see that we need to *extend* this to 2×2 $\times 3 \times 3 \times 7$ (to "cover" 84). So $2 \times 2 \times 3 \times 3$ $\times 7$ or 252 is the lowest whole number that is a multiple of both 36 and 84.

25 and 21 are relatively prime because 25 = 5 x 5 and 21 = 3 x 7, so there are no prime factors in common. 25 and 30 are not relatively prime because 25 = 5 x 5 and 30 = 2 x 3 x 5, so they have a "5" in common. 26 and 13 are not relatively prime because they each have 13 as a factor.

12 can be thought of as 2×6 (2 rows of 6) or 3×4 or 1×12 .

Venn Diagram: A visual way of organizing information logically to show what is included or excluded from a definition.

See Prime Time Homework Examples from ACE on this website. (Investigation 2 #17)