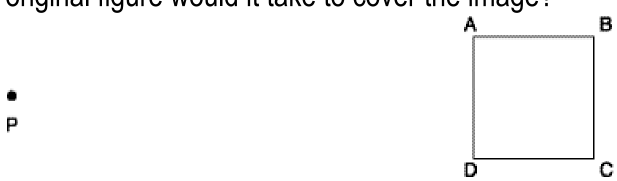
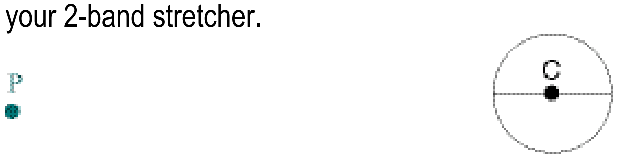
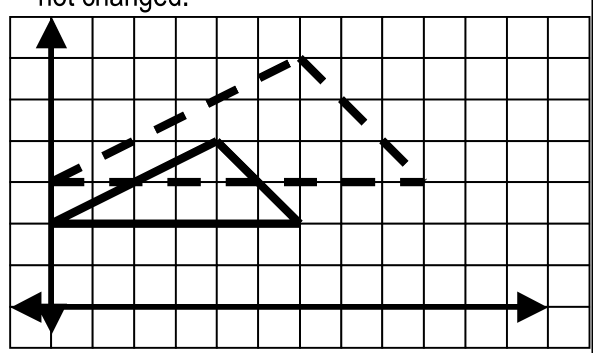
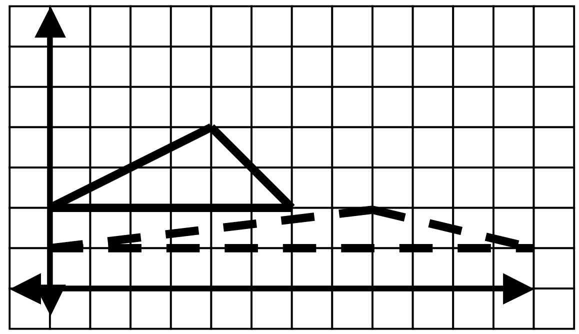
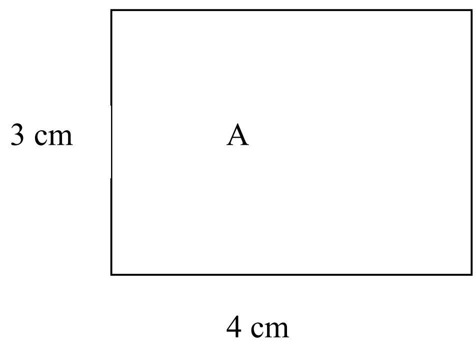
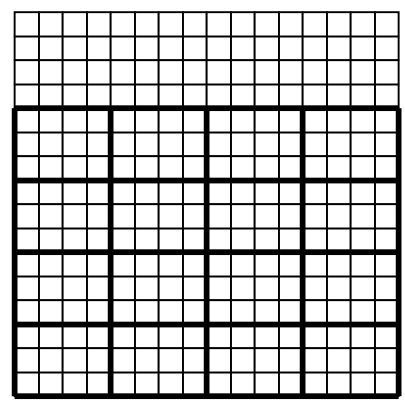
Stretching and Shrinking: Homework Examples from ACE

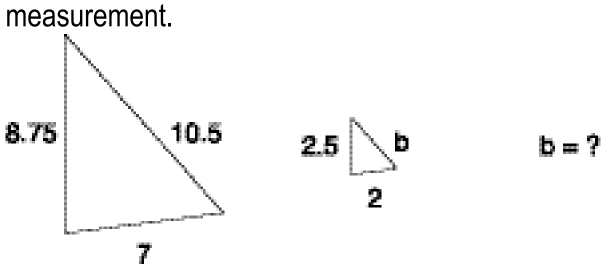
ACE Investigation 1: #3, 13.
ACCE Investigation 2: #3, 10.
ACE Investigation 3: #19. 29. 30.
ACE Investigation 4: 6, 38.
ACE Investigation 5: #5.

ACE Question	Possible Answer
ACE Investigation 1	
3. Copy the figure ABCD and anchor point P onto a sheet of paper, and enlarge the figure with a two-band stretcher. Then answer parts a – d. a. How do the side lengths of the original figure compare to the side lengths of the image? b. How does the perimeter of the original figure compare to the perimeter of the image? c. How do the angle measures of the original figure compare to the angles measures of the image? d. How does the area of the original figure compare to the area of the image? How many copies of the original figure would it take to cover the image?  	3. a. The side lengths should be twice as long. (This is because we used a 2 band stretcher. If we used a 3 band stretcher the image lengths would be three times as long. Students may not realize that it is because the distances from the *image to the anchor point* are all twice as long as the corresponding distances from the *original to the anchor point* that the image is twice as large. There are, in fact, similar triangles embedded in the drawing.) b. The perimeter should also be twice the original. (This makes sense since each length has been doubled, so the sum of all the sides will also have been doubled.) c. The angles are still the same, 90 degrees. d. The area has been increased by a factor of 4. (This is often a surprise to students who think that if the lengths are doubled the area should be doubled also. You can convince a student of this by drawing a square with sides of 1 inch, and another square with sides of 2 inches. *Four* of the smaller squares fit inside of the larger square.)
13. Copy the circle C and anchor point P onto a sheet of paper. Make an enlargement of the circle using your 2-band stretcher.  a. How do the diameters of the circles compare? b. How do the areas of the circles compare? c. How do the circumferences of the circles compare?	13. a. The image has a diameter that is twice as large as the original. (The position of the anchor point is not relevant, but students may think that an anchor point further from the circle causes more stretch in the band and so more enlargement. However, the important idea is not how much a rubber band stretches, but that there are 2 bands and they stretch equally, making the *image twice as far from the anchor point as the original, and,*

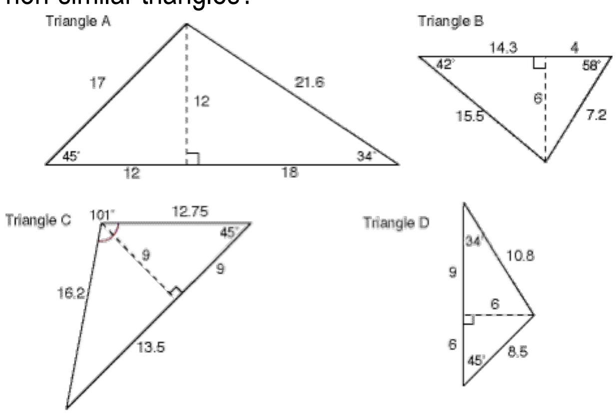
	therefore, twice as big.) b. The area of the image circle is 4 times as large as the original. (Student drawings may be quite inaccurate, but if they can measure the diameters of the original and the image they can calculate the areas, using Area = pi(radius squared). They can also try to find the area by covering it with a grid and counting squares.) c. The circumference of the image is twice the circumference of the original. This is because the circumference = pi(diameter). So if the diameter has doubled then so has the circumference.
ACE Investigation 2	
3. a. On grid paper, draw a triangle ABC with vertex coordinates A(0, 2), B(6, 2) and C(4, 4). b. Apply the rule (1.5x, 1.5y) to the vertices of triangle ABC to get triangle PQR. Compare the corresponding measurements (side lengths, perimeter, area, angle measures) of the two triangles. c. Apply the rule (2x, 0.5y) to the vertices of the triangle ABC to get triangle FGH. Compare the corresponding measurements (side lengths, perimeter, area, angle measures) of triangle ABC and FGH. d. Which triangle, PQR or FGH , seems similar to triangle ABC? Why?	3. a and b. (0,2) becomes (0, 3). (6,2) becomes (9, 3). (4, 4) becomes (6, 6). The sides of the image are 1.5 times as long as the original. The perimeter is 1.5 times as long. The area of the original triangle, using the formula 0.5(base)(height), is (0.5)(6)(2) = 6 square units. The area of the image triangle is (0.5)(9)(3) = 13.5. So the new area is 2.25 times the original. The angle measures have not changed.  c. (0, 2) becomes (0, 1). (4, 4) becomes (8, 2), (6, 2) becomes (12, 1). The length of FG is twice the length of AB. The length of FH is more than, but not twice, AC. (Actually if you apply the Pythagorean theorem we can find the lengths, but students have not studied this yet, so they will have to rely on estimation.)

	Likewise the length of GH is more than but not twice BC. The area of triangle ABC is 6 square units. The area of triangle FGH is (0.5)(12)(1) = 6 square units. (this makes sense. We doubled one dimension and halved the other so the area remains unchanged.) The angles are quite changed.  d. Angles in triangle PQR are equal to corresponding angles in ABC, and lengths in PQR are double the corresponding lengths in ABC. So triangle PQR is similar to triangle ABC. Note: similar figures have the same shape (corresponding angles must be the same size), and are scale copies of each other (corresponding sides must be in the same ratio).
10. What is the scale factor from an original figure to its image if the image is created by using a. a 2-rubber-band stretcher b. a copy machine with size factor 150% c. a copy machine with size factor 250% d. the coordinate rule (0.75x, 0.75y)	10. a. The scale factor will be 2. (Ratio of length in image: corresponding length in original = 2:1. Also, distance from a point on the image to the anchor point is twice the distance of the corresponding point on the originalto the anchor point.) b. 1.5 (Ratio of length in image: corresponding length in original = 150:100 or 1.5:1) c. 2.5 d. 0.75 (Ratio of length in image: corresponding length in original = 0.75:1)
ACE Investigation 3	
19. Suppose a rectangle B is similar to rectangle A below. If the scale factor from rectangle A to rectangle B is 4, what is the area of rectangle B?	19. A scale factor of 4 (from A to B) means that lengths of rectangle A have to be multiplied by 4

	to make the lengths of rectangle B. Thus, the sides of rectangle B are 3(4) cm and 4(4) cm. This means that the area of rectangle B is 12 x 16 square centimeters or 192 square centimeters. Note: Rectangle B is 16 times as large as Rectangle A. (In investigation 3 students combined multiple copies of specific shapes to create a larger, similar shape. The original shapes were named *rep-tiles*. In this case, rectangle A is a *rep-tile* and 16 copies of the rectangle would be needed to create rectangle B.) 
29. True or False? All squares are similar.	29. For two shapes to be similar their corresponding angles have to be the same size, and corresponding sides have to be in the same ratio. Angles: All angles of a square are right angles, so this condition is fulfilled. Sides: Suppose one square has sides of 2 units, and another has sides of 5 units. Then the ratio of corresponding sides is 5/2, for all pairs of corresponding sides. If the squares had sides x and y respectively, then the ratio of corresponding sides would be y/x for all corresponding pairs. So this condition is fulfilled, no matter what size the squares are.

	True: all squares are similar.
30. True or False? All rectangles are similar.	30. Angles: All angles in a rectangle are 90 degrees. Therefore, corresponding angles in 2 rectangles will be the same size. Sides: Suppose one rectangle is 3 cm by 4 cm, and another is 6cm by 7 cm., then the ratio of the short sides is 6/3, and the ratio of the long sides is 7/4. These are not equal ratios; therefore, the rectangles are not similar. False: All rectangles are NOT similar. Note: we only need ONE counterexample to prove an assertion is false. BUT one example or even many examples are not sufficient to prove some statement is always true.
ACE Investigation 4	
6. The triangles are similar. Find the missing measurement.  	6. There are several ways to do this problem. **Scale factor**: Comparing corresponding sides, the scale factor from large to small is 2/7. Therefore, we have to multiply lengths in the large triangle by 2/7 to find corresponding lengths in the small triangle. B = (2/7)(10.5) = 3. **Ratios between the triangle**: Comparing lengths of corresponding sides we have 2.5/8.75 = b/10.5 One way to find b is to rewrite these fractions with a common denominator. Simplifying first we have, 2.5/8.75 is the same as 10/35, and b/10.5 is the same as 2b/ 21, so 10/35 = 2b/21, so 30/105 = 10b/105, so b = 3. **Ratios within each triangle**:

<table>
<tr><td></td><td>10.5/7 = b/2.
We could proceed as above, or we could compute 10.5/7 = 1.5, and write
1.5 = b/2.
So b = 3.</td></tr>
<tr><td>38.

a. Identify the triangles that are similar to each other.
b. For each triangle, find the ratio of base to height. How do these ratios compare for the similar triangles? How do these ratios compare for the non-similar triangles?

</td><td>38.
a. Since we know that the angles of any triangle sum to 180 degrees (Shapes and Designs) we can find the missing angles in each triangle. Therefore, we know that the corresponding angles in triangles A, C, D are equal. (But not the angles in triangle B.) So triangles A, C, D are similar. (We do not have to check the scale factor for corresponding sides, but if we do we find that the scale factor from A to C is 12.75/17 or 16.2/21.6 or 22.5/30, which is 0.75 in each case. Likewise the scale factor from A to D is 0.5.)
b.
For triangle A,
Base/height = 30/12 = 2.5.
For triangle B,
Base/height = 10.4/6 = 1.733
For triangle C,
Base/height = 22.5/9 = 2.5.
For triangle D,
Base/height = 15/6 = 2.5.

This is a confirmation that for similar triangles a ratio made of sides or lengths within one triangle will be equal to a ratio made of corresponding sides or lengths within the other triangle.</td></tr>
<tr><td>Investigation 5</td><td></td></tr>
<tr><td>5.
Judy lies on the ground 45 feet from her tent. Both the top of the tent and the top of a tall cliff are in her line of sight. Her tent is 10 feet tall. About how high is the cliff? How do you know?</td><td>5.
There is a pair of similar triangles in this figure. We know the triangles are similar because we know that corresponding angles in the two triangles are equal; we can see that each triangle has a right angle and also the triangles share an angle (at Judy's head).</td></tr>
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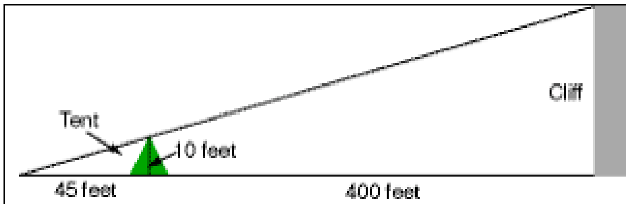

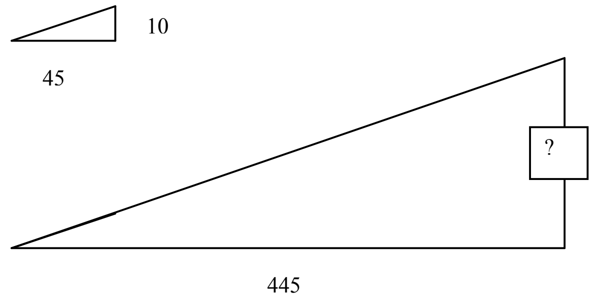

The scale factor from small to large is 445/45. Therefore, to find the height of the cliff we need to multiply 10 by the same scale factor. The cliff height is 10(445/45) = 98.9 feet approximately.

(Alternative solution strategies might involve setting up equal ratios:
cliff/10 = 445/45 etc.)

Note: we only need to know that corresponding angles in two triangles are equal to deduce that the triangles are similar. We can then figure out missing lengths. BUT, knowing that corresponding angles are equal is NOT enough information to show that other figures are similar. For example, see Investigation 3 #30.