Variable and Patterns: Homework Examples from ACE
ACE Investigation 1: #5.
ACE Investigation 2: #15.
ACE Investigation 3: #13 – 16, #17 – 19.
ACE Investigation 4: #4.

	Possible Answers

ACE Investigation 1

5. Below is a chart of the water depth in a harbor during a typical 24-hour day. The water level rises and falls with the tides.

Hours since midnight	0	1	2	3	4	5	6
Depth (meters)	10.1	10.6	11.5	13.2	14.5	15.5	16.2

Hours since midnight	7	8	9	10	11	12
Depth (meters)	15.4	14.6	12.9	11.4	10.3	10.0

Hours since midnight	13	14	15	16	17	18
Depth (meters)	10.4	11.4	13.1	14.5	15.4	16.0

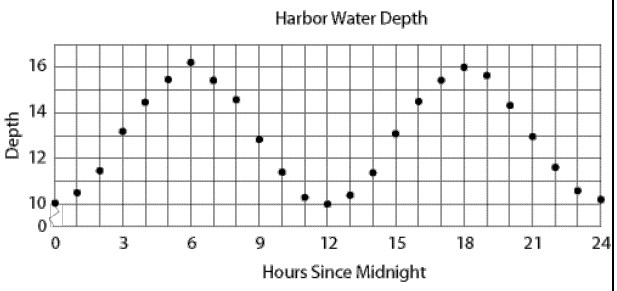
Hours since midnight	19	20	21	22	23	24
Depth (meters)	15.6	14.3	13.0	11.6	10.7	10.2

a. When is the water the deepest? What is the depth at that time?

b. When is the water the shallowest? What is the depth at that time?

c. During what time interval does the water depth change

5. a. The water is deepest at 6 hours after midnight, or 6:00 a.m., with a depth of 16.2 m.
   b. The water is shallowest at noon with a depth of 10.0 m.
   c. The water depth changes most rapidly—by 1.7 meters—during each of these hours: from 2 to 3 (2 a.m.–3 a.m.), from 8 to 9 (8 a.m.–9 a.m.), and from 14 to 15 (2 p.m.–3 p.m.).
   d. The pattern of the graph is bimodal (two humps). It looks symmetric, so that if it was flipped over when $x = 12$ (hour 12), the two parts would line up. Overall, the graph rises to hour 6, then the water depth goes back down, and then rises again to hour 18, and then the depth decreases again.



e. Possible answer: I used 1-hour intervals on the *x*-axis because these were the time intervals given in the table. I used 2-meter intervals on the *y*-axis because it allowed all the data to be graphed on my grid paper. (Not all students will use this scale. They might use 1 meter intervals on the vertical axis, because the numbers range from 10 to 16.2, not a large range. Or they might want to use 0.5 meter intervals or even smaller, trying to show the decimal numbers more accurately. It depends on how much room they have vertically. They do not have to show the numbers 0 – 9 on the vertical axis since these are not used, but if they omit these then they must indicate that this has been done, as above. They should not simply mark 0 then 10 on this axis. *Above all, increments on the axes must have the same values, with tick marks every 1 or every 2 or every 0.5 meter, for example. A common error is to mark the vertical axis with the*

most rapidly?

d. Make a coordinate graph of the data. Describe the overall pattern you see.

e. How did you determine what scale to use? Do you think everyone in your class used the same scale?

numbers given in the table.)

ACE Investigation 2

15. The area of a rectangle is the product of its length and its width.

a. Find all whole number pairs of length and width values that give an area of 24 square meters. Record the pairs in a table.

b. Make a coordinate graph of your data from part a. Put length on the *x*-axis and width on the *y*-axis.

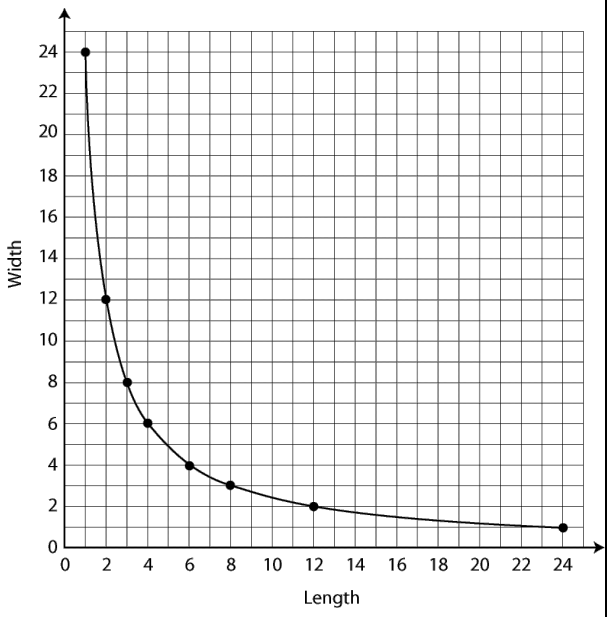
c. Connect the points if it makes sense to do so. Explain your decision.

d. Describe the relationship between length and width for rectangles of area 24 square meters.

15. a. (Notice the connection here with factors from Prime Time.)

Length	Width
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

b.



c. There are possible lengths and widths between the whole numbers shown in the table. It makes sense to connect the points from the table to show where these other points will lie.

d. Possible answer: As the length increases the width decreases, rapidly when length is small, and then more slowly as length gets larger.

ACE Investigation 3

13. The sales tax in a state is 8%. Write an equation for the amount of tax, t, on an item that costs p dollars.

14. An airplane travels at an air speed of 550 miles per hour. Write an equation for the distance d the plane travels in h hours.

15. Potatoes sell for $0.25 per pound in a market. Write an equation for the cost c of p pounds of potatoes.

16. A cellular phone plan costs $49 per month plus $0.05 per minute of long-distance. Write an equation for the monthly bill b when m minutes of long distance service are used.

For 17 – 19, describe the relationship between the variables in words and with an equation.

17.

x	1	2	5	10	20
y	4	8	20	40	80

18.

s	1	2	3	6	12
t	49	48	47	44	38

19.

n	1	2	3	4	5
z	6	11	16	21	26

One strategy for finding an equation is to make a table with a few pairs that you can work out from the given information, then look for a pattern that you can continue in the table. This gives you a way of checking that any equation you propose does in fact fit the pattern.

13. Say we made a purchase of $1.00 then the tax is $0.08, for $2.00 the tax is $0.16 etc. In a table this is

Purchase $p	1	2	3	4
Tax. $T	0.08	0.16	0.24	0.32

T = 0.08p.

14. $d = 550h$

15. $C = 0.25p$

16. Say we talk for 1 minute then the Bill is $49 + $0.05, for 2 minutes, $49 + $0.10 etc. In a table this is

Minutes, m	1	2	3	4
Bill, $B	49.05	49.10	49.15	49.20

Students may find the two bits of information distracting and want to try B = 49m or B = 0.05m, neither of which produces the pairs in the table. To get the pairs in the table we hold the $49 constant, no matter how many minutes and change the amount added as m changes.

$B = 0.05m + 49$

17. $y = 4x$

18. Students will notice that the t values decrease by 1 as s increases by 1. They may try t = 49s, if they only look at the first pair. They may try 49s – 1 or 49 – s or other variations, as they try to think out how "49" and "-1" combine to produce these pairs. If the y-intercept were

given (0,50) this would be an additional clue that helps. $t = 50 - s$.

19. Students will observe that the values of z increase by a constant rate of 5 for each increase of 1 in n. Again, if the y-intercept is given (or worked out, by working backwards) then the pair (0, 1) would be an additional clue. $z = 5n + 1$

ACE Investigation 4

4. The operators of *Ocean Bike Tours* consider leasing a small bus. They compare two companies. Superior Buses charges $5 for each mile driven. East Coast Transport charges $1000, plus $2.50 per mile driven.

a. For each company, write an equation to show how the *bus lease cost*, *C*, depends on *number of miles driven*, *m*.

b. Enter both equations into your calculator. Choose window settings that make sense for this situation and that show a good view of both graphs. Sketch the graphs, and tell what axes limits (X Min, XMax, YMin, Y Max) you used.

c. Use the TRACE feature of your calculator to estimate coordinates of the point at which the lease cost is the same for both companies. Explain what the coordinates of the point tell you about the bus rental situation.

d. For what driving mileage would the East Coast lease be a better deal? For what mileage would the Superior Bus lease be better? Explain how your answers are shown on the graph in part b.

4. Students find using calculators very helpful in moving quickly from equation to the more concrete forms of a table or graph, and then reading specific information from these other representations. The initial learning about how to set up a table and graph is slow, but once students realize the questions they have to ask themselves to adjust a table or graph to see relevant information they quickly become sophisticated users of this technology. Parents and guardians are likely to learn more slowly than their children and will probably have to rely on them for explanations.

a. East Coast cost = 1000 + 2.5*m*; Superior cost = 5*m*

b. Student answers will vary. One strategy is to make a calculator table first, since it requires less set up. This will produce

Number of miles, m	Cost for East Coast, $C	Cost for Superior, $C
0	1000	0
10	1025	50
20	1050	100
…	…	…
400	2000	2000
410	2025	2050
…		

By scrolling down the table we see that for

	fewer that 400 miles Superior is cheaper, and for more than 400 miles East Coast is cheaper. Thus, the interesting part of the graph will be around (400, 2000). The x-range has to include 400; the y range has to include 2000, so a scale has to make these large values appear on the screen. NOTE: even if the actual intersection does not appear in the table because it occurs between pairs produced by the choice of table increments, students should be able to see the approximate point when one Cost changes from being less to being more than the other. One possible choice for a window setting is: Xmin = 0 Xmax = 1000 Xscl = 100 Ymin = 0 Ymax = 5000 Yscl = 500 c. (400, 2000) At this point we see that the cost is $2000 for 400 miles, for both companies. NOTE: Tracing is unlikely to land the cursor exactly on this point, because the cursor jumps from pixel to pixel. The table is easier to manipulate to choose increments to land exactly on the point of intersection. d. East Coast is a better deal when mileage is over 400 miles. Superior is a better deal when mileage is less than 400 m.