# Let's Be Rational: Homework Examples from ACE

Investigation 1: *Extending Addition and Subtraction of Fractions*, ACE #7, #13, #20, #53
Investigation 2: *Building on Multiplication of Fractions*, ACE #2, #17, #23, #37
Investigation 3: *Dividing with Fractions*, ACE #1, #17, #18, #19, #34
Investigation 4: *Wrapping Up the Operations*, ACE #5, #6, #7, #28

Investigation 1: *Extending Addition and Subtraction of Fractions*
ACE #7

For Exercises 7-12, determine whether the sum of the two Getting Close game cards is closest to 0, 1, 2. or 3. Explain.

7. $\frac{7}{8}$ and $\frac{4}{9}$.

7. 1. Possible Explanation: $\frac{7}{8}$ is a little less than 1 and $\frac{4}{9}$ is a little less than $\frac{1}{2}$.

Together, a little less than 1 and a little less than $\frac{1}{2}$ is a little less than $1\frac{1}{2}$, which is closer to 1 than 2.

Investigation 1: *Extending Addition and Subtraction of Fractions*
ACE #13

For Exercises 13-15, you are playing a game called Getting Even Closer. In this game, yo have to estimate sums to the nearest ½ or 0.5. Decide if the sum of the two game cards turned up is closest to 0, 0.5, or 1. Explain.

13. The two cards shown are $\frac{3}{5}$ and $\frac{1}{10}$.

13. 0.5; Possible Explanation: $\frac{3}{5}$ is a bit more than half, and $\frac{1}{10}$ is a small fraction, but not big enough to push $\frac{3}{5}$ closer to 1 when the two are added together.

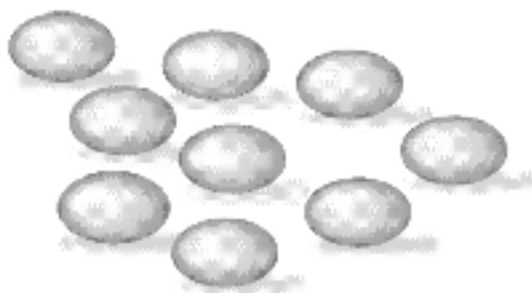
Investigation 1: *Extending Addition and Subtraction of Fractions*
ACE #20

For Exercises 17-20, find two fractions with a sum that is between the two given numbers.

20. $1\frac{1}{2}$ and 2

20. There are many answers to this. One way to think about this problem is that because $\frac{3}{4}+\frac{3}{4}=1\frac{1}{2}$ and 1 + 1 = 2, and because we want the sum to be between $1\frac{1}{2}$ and 2, we might choose two fractions between $\frac{3}{4}$ and 1. We can make one fraction a little more than $\frac{3}{4}$ and the other a little less than 1. For example, $\frac{7}{8}+\frac{9}{10}$.

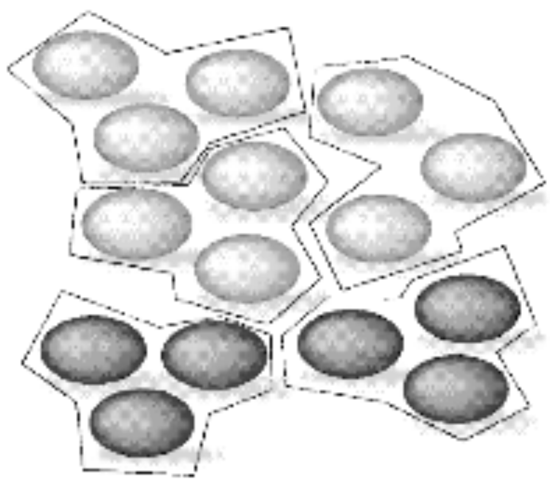
53. The beans show represent $\frac{3}{5}$ of the total beans on the kitchen counter. How many total beans are there on the counter?



53. Solution:

Because 9 beans represents $\frac{3}{5}$ of the total, each group of 3 beans must be $\frac{1}{5}$ of the total. We need $\frac{2}{5}$ more to make the total, so we need 6 more beans.

The picture below helps to describe our thinking.

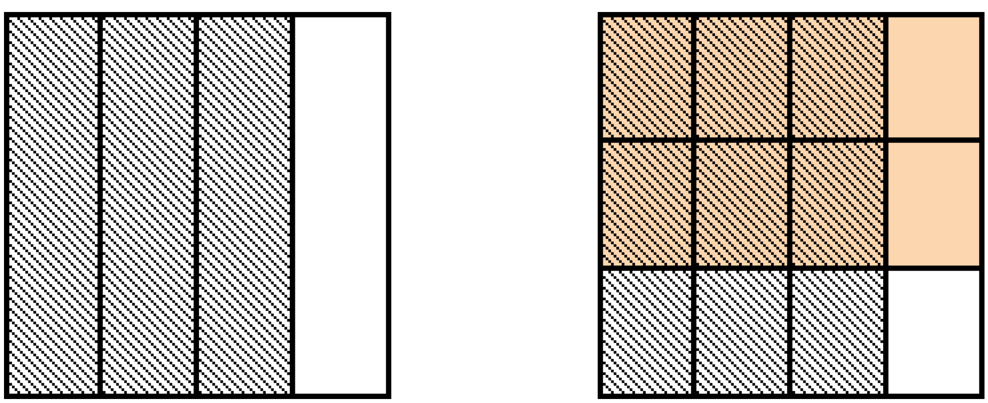


Investigation 2: *Building on Multiplication of Fractions*
ACE #2

2. a. Draw brownie-pan models to show whether or not $\frac{2}{3}$ of $\frac{3}{4}$ of a pan of brownies is the same amount as $\frac{3}{4}$ of $\frac{2}{3}$ of a pan of brownies.
   b. If the brown pans are the same size, how do the amounts of brownies from part a compare?
   c. Describe the relationship between $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{2}{3}$.

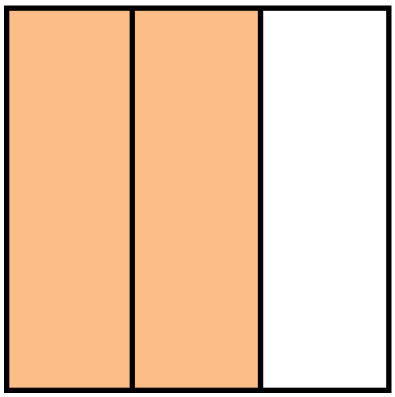
2. The "brownie pan model" is actually an area model and refers to problems students investigated in class.

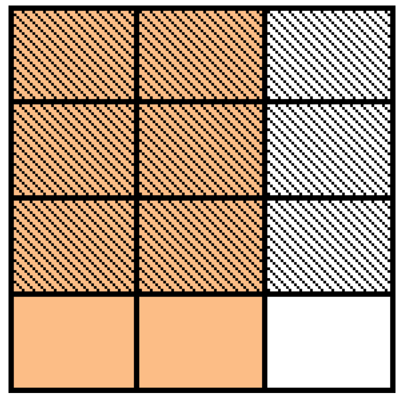
a. First identify $\frac{3}{4}$ of a rectangular area by drawing vertical lines and cross-hatching $\frac{3}{4}$ of the vertical strips. Next, divide this into thirds by drawing horizontal lines and lightly shading $\frac{2}{3}$ of the horizontal strips (see the diagram below). The overlapping shaded and cross-hatched area is $\frac{2}{3}$ of $\frac{3}{4}$. This area is $\frac{6}{12}$.



(Note that students may also notice that there are 3 vertical strips representing $\frac{3}{4}$ of the pan, so $\frac{2}{3}$ would be 2 of these vertical strips, or $\frac{2}{4}$. This also shows that $\frac{2}{4}$ is equivalent to $\frac{6}{12}$.)

Second, Identify $\frac{2}{3}$ of a rectangular area by drawing vertical lines and shading $\frac{2}{3}$ of the vertical strips. Next, divide this into fourths by drawing horizontal lines and cross-hatching $\frac{3}{4}$ of the horizontal strips (see the diagram below). The overlapping cross-hatched and shaded area is $\frac{3}{4}$ of $\frac{2}{3}$. This area is $\frac{6}{12}$.





Note that for each procedure, we get the same area as a result.

b. Depending on how students draw this area model, the first calculation will result in $\frac{6}{12}$ which equals $\frac{1}{2}$. The second area model and calculation also results in $\frac{6}{12}$. *This model leads to the common shortcut of multiplying the numerators and denominators.*

c. Student should be able to connect this idea about multiplying fractions to what they know about multiplying whole numbers: that multiplication is commutative. The reason is perhaps not immediately obvious due to the use of the word “of” in these initial examples of fraction multiplication.

Investigation 2: *Building on Multiplication of Fractions*
ACE #17

17. Find each product. Look for patterns to help you.

a. $\frac{1}{3} \times 18$

b. $\frac{2}{3} \times 18$

c. $\frac{5}{3} \times 18$

d. $1\frac{2}{3} \times 18$

17. a. Students might draw 18 objects and group these into three equal-sized groups of 6. So, $\frac{1}{3}\ of\ 18$ = 6.

b. If $\frac{1}{3}\ of\ 18$ = 6, then $\frac{2}{3}\ of\ 18$ should be twice as much, or 12.

c. If $\frac{1}{3}\ of\ 18$ = 6, the $\frac{5}{3}\ of\ 18$ it 5 times as much, or 30.

d. If students think of $1\frac{2}{3}$ as the same as $\frac{5}{3}$ they will get the same answer as in part c. Or they may think of this as $1 \times 18 + \frac{2}{3} \times 18$ and get 18 + 12 = 30.

Investigation 2: *Building on Multiplication of Fractions*
ACE #23

For Exercises 19-27, use an algorithm for multiplying fractions to determine each product.

23. $10\frac{3}{4} \times 2\frac{2}{3}$

23. Students might do this by renaming each factor as $\frac{43}{4} \times \frac{8}{3}$ and then multiplying the numerators and denominators, to get $\frac{344}{12} = 28\frac{8}{12}$.

OR

They might think of $\frac{1}{4}$ *of* $\frac{8}{3} = \frac{2}{3}$ and so $\frac{43}{4}$ must be 43 times as much, or $43\left(\frac{2}{3}\right) = \frac{86}{3} = 28\frac{2}{3}$ (as before).

OR

They might think of this as

$$10\left(2\frac{2}{3}\right) + \frac{3}{4}\left(2\frac{2}{3}\right)$$
$$= 10\left(2\frac{2}{3}\right) + \left(3\,times\,\frac{1}{4}\,of\,\frac{8}{3}\right)$$
$$= \left(20 + \frac{20}{3}\right) + 3\left(\frac{2}{3}\right)$$
$$= 20 + \frac{20}{3} + \frac{6}{3}$$
$$= 20 + 6\frac{2}{3} + 2$$
$$= 28\frac{2}{3}$$

Students have various ways to think about multiplication of mixed numbers, and all the correct ways involve logical steps, or algorithms, that they might use to solve these problems.

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| Investigation 2: *Building on Multiplication of Fractions*<br>ACE #37 |

37. Multiple Choice: Which of the numbers below, when multiplied by $\frac{4}{7}$ will be less than $\frac{4}{7}$?

a. $\frac{1}{7}$ b. $\frac{7}{7}$ c. $\frac{17}{7}$ d. $\frac{8}{7}$

37. The goal for this set of multiple-choice problems (#36-38) is for students to use mathematical thinking rather than calculating the problem out. In class, students have been working on generalizing patterns in the multiplication of rational numbers. For this problem, students need to understand that to get a result less than $\frac{4}{7}$ we must multiply by any number that is less than 1. For example, $\frac{9}{10}\times\frac{4}{7}=\frac{36}{70}$. From the available choices we know that choice a, multiplying $\frac{4}{7}$ by $\frac{1}{7}$, will produce an answer less than $\frac{4}{7}$ (note that choice d. is different here than in #36 & #38).

Investigation 3: *Dividing with Fractions*
ACE #1

1. A latte is the most popular drink at Antonio's Coffee Shop. Antonio makes only one size of latte, and he uses $\frac{1}{3}$ cup of milk in each drink. How many lattes can Antonio make with the amount of milk in containers (a)-(c)? If there is a remainder, what does it mean?

   a. $\frac{7}{9}$ b. $\frac{5}{6}$ c. $3\frac{2}{3}$

1.

a. This asks how many times $\frac{1}{3}$ goes into $\frac{7}{9}$, or $\frac{7}{9} \div \frac{1}{3}$**.** One way to do this is to rename the fraction $\frac{1}{3}$ as $\frac{3}{9}$**.** We know that $\frac{7}{9} \div \frac{1}{9}$ would be 7, so $\frac{7}{9} \div \frac{3}{9}$ would equal a third as much, or $\frac{7}{3}$ , which is **$2\frac{1}{3}$ lattes**. A common error of interpreting this result is for students to think that $\frac{1}{3}$ is the remainder, but that is not the case. The result of the division, $2\frac{1}{3}$ lattes. says that **Antonio can make 2 lattes**, using $\frac{2}{3}$ , or $\frac{6}{9}$ , of a cup of milk, **with $\frac{1}{9}$ *of a cup of milk* left over**, which is enough to make only $\frac{1}{3}$ *of a latte*.

b. This asks how many times $\frac{1}{3}$ goes into $\frac{5}{6}$, or $\frac{5}{6} \div \frac{1}{3}$**.** One way to do this is to rename the fraction $\frac{1}{3}$ as $\frac{2}{6}$**.** We know that $\frac{5}{6} \div \frac{1}{6}$ would be 5, so $\frac{5}{6} \div \frac{2}{6}$ would equal half as much, or $\frac{5}{2}$ , which is $2\frac{1}{2}$ lattes. This says that Antonio can make 2 lattes, using $\frac{2}{3}$ or $\frac{4}{6}$ of a cup of milk, with $\frac{1}{6}$ *of a cup of milk* left over, which is enough to make $\frac{1}{2}$ *of a latte*.

c. This asks how many times $\frac{1}{3}$ goes into $3\frac{2}{3}$, or $3\frac{2}{3} \div \frac{1}{3}$**.** One way to do this is to rename the fraction $3\frac{2}{3}$ as $\frac{11}{3}$**.** We know that $\frac{11}{3} \div \frac{1}{3}$ would be 11. So with $3\frac{2}{3}$ cups of milk, Antonio can make 11 lattes. There is no remainder, so there is no milk left over.

Investigation 3: *Dividing with Fractions*
ACE #17

For Exercises 17-19, find each quotient. Draw a picture to prove that each quotient makes sense.

17. $\frac{4}{5} \div 3$

Solution

The 2 ways of thinking about division problems:

There are two ways to think about division. The drawing that students choose will depend on how they think about the problem.

a. We might think this problem says, "We have $\frac{4}{5}$ of something and we want to divide it up in to three pieces. How big is each piece?" This is the partition way of thinking about division.

OR

b. We might think of this as, "We have $\frac{4}{5}$ of something and we want to find out how many times 3 will go into that." This the grouping way of thinking about division. Obviously 3 is too big to make even one group of 3, so the question becomes, "What part of 3 will go into $\frac{4}{5}$ ?"

Solution to #17:

Thinking of this as partition, we need to find a way to break $\frac{4}{5}$ into 3 pieces. One way to do this would be to divide each $\frac{1}{5}$ into 3 pieces. These would be fifteenths, and there would be $\frac{12}{15}$ shaded in the model below. Then we can share these fifteenths into three groups; each group has a size of $\frac{4}{15}$ .

Investigation 3: *Dividing with Fractions*
ACE #18 & 19

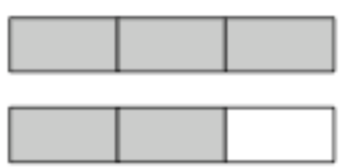
For Exercises 17-19, find each quotient. Draw a picture to prove that each quotient makes sense.

18. $1\frac{2}{3} \div 5$
19. $\frac{5}{3} \div 5$

Solution
See the introduction to #17 above, which discusses the two ways to think about division, partition and grouping.

18. If we think of this as "How many times will 5 fit into $1\frac{2}{3}$?" we have to say that is the group size is 5, then only $\frac{1}{3}$ of the group size will fit into $1\frac{2}{3}$.

If we think of this as $\frac{5}{3}$ to be divided into 5 pieces and asking "How big is each piece?" we can see that each piece is $\frac{1}{3}$.



19. $1\frac{2}{3} = \frac{5}{3}$. See #18 above.

Investigation 3: *Dividing with Fractions*
ACE #34

34. Find the quotient.

$$2\frac{1}{2} \div 1\frac{1}{3} = \frac{5}{2} \div \frac{4}{3}$$

34. Students may do this by renaming with common denominators:

$$\begin{aligned} 2\frac{1}{2} \div 1\frac{1}{3} &= \frac{5}{2} \div \frac{4}{3} \\ &= \frac{15}{6} \div \frac{8}{6} \\ &= \frac{15}{6} \div \frac{8}{6} \end{aligned}$$

Students might think about this last step as $\frac{15}{6} \div \frac{1}{6} = 15$, so $\frac{15}{6} \div \frac{8}{6}$ will be an eighth of this, or $\frac{15}{8}$ .

*Note that this reasoning leads to the common shortcut of multiplying by the denominator and dividing by the numerator of the second fraction in a division problem, which is commonly called "when dividing by a fraction, multiply by the reciprocal." The goal is for students to understand why this works.*

| Investigation 4: *Wrapping Up the Operations*, ACE #5 |
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| Find the value of N that makes each number sentence correct.<br>5. $\frac{2}{3} + \frac{3}{4} = N$<br><br>5. Students may make a sketch of an area model or use a number line solve this problem. Whatever strategy they use, they will have to rename the two fractions with a common denominator. $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$ |

| Investigation 4: *Wrapping Up the Operations*, ACE #6 |
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| Find the value of N that makes each number sentence correct.<br>6. $\frac{3}{4} + N = \frac{4}{5}$<br><br>6. Students use "fact families" to rephrase this number sentence as $\frac{4}{5} - \frac{3}{4} = N$. Renaming the fractions with a common denominator of 20, we have:<br>$N = \frac{4}{5} - \frac{3}{4} = \frac{16}{20} - \frac{15}{20} = \frac{1}{20}$ |

| Investigation 4: *Wrapping Up the Operations*, ACE #7 |
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| Find the value of N that makes each number sentence correct.<br>7. N $-\frac{3}{5} = \frac{1}{4}$<br><br>7. Students use "fact families" to rephrase this number sentence as $\frac{1}{4} + \frac{3}{5}$ = N.<br>Renaming the fractions with a common denominator of 20, we have:<br>N = $\frac{1}{4} + \frac{3}{5} = \frac{5}{20} + \frac{12}{20} = \frac{17}{20}$ |

| Investigation 4: *Wrapping Up the Operations*, ACE #28 |
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| 28. Kalin walks at a steady rate of 3 $\frac{2}{3}$ miles per hour. The beach is 4 $\frac{1}{4}$ miles from his home. How long will it take Kalin to walk from his home to the beach and back to his home?<br><br>28. One solution is to calculate the time one way by dividing the distance by the rate, and then multiply that by two.<br>$4\frac{1}{4} \div 3\frac{2}{3}$<br>$= 4\frac{3}{12} \div 3\frac{8}{12}$<br>$= \frac{51}{12} \div \frac{44}{12}$<br>$= \frac{51}{44} = 1\frac{7}{44}$ hours<br>So one way it takes $1\frac{7}{44}$ hours (which is about 1 hour and 10 minutes).<br>Mutliplying that by two, to get the time for the two way trip, we get $2\frac{7}{22}$ hours (which is about 2 hours and 19 minutes). |