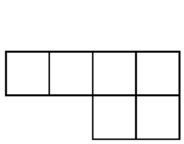
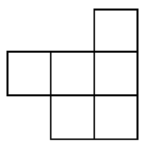
Covering and Surrounding: Homework Examples from ACE

Investigation 1: *Extending and Building on Area and Perimeter,* ACE #4, #6, #17
Investigation 2: *Measuring Triangles,* ACE #4, #9, #12
Investigation 3: *Measuring Parallelograms,* ACE #7
Investigation 4: *Measuring Surface Area and Volume,* ACE #17, #22

Investigation 1: *Extending and Building on Area and Perimeter*
ACE #4

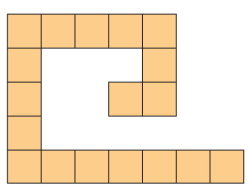
Draw two different shapes, each with an area of 6 square units and perimeter of 12 units.

This question reinforces the idea that the relationship between perimeter and area is not simple.
If students experiment with square tiles they can come up with arrangements of 6 square tiles with a perimeter of 12 units. Here are two examples:

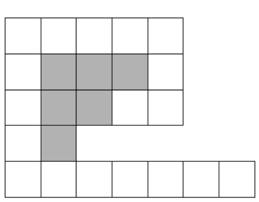



Investigation 1: *Extending and Building on Area and Perimeter*
ACE #6

Copy the design below onto grid paper. Add six squares to make a new design with a perimeter of 30 units. Explain how the perimeter changes as you add new tiles to the figure.



We need to reduce the perimeter while increasing the area. Students learned that more "compact" figures can cover the same area without increasing the perimeter. This idea means that they have to "fill in" some of the blank space in the center of the shape.



Adding these six tiles reduced the perimeter of the figure. Only two of the new tiles have exposed edges, while together they cover ten previously exposed edges in the original figure.

Perimeter = 7 +1 +5 +1 +3 +3 +5 +5 = 30.

Investigation 1: *Extending and Building on Area and Perimeter*
ACE #17

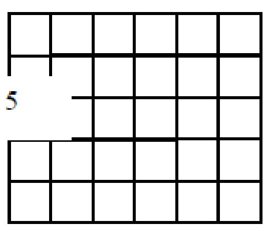
Copy and complete the table. Sketch each rectangle and label its dimensions.

Rectangle Area and Perimeter

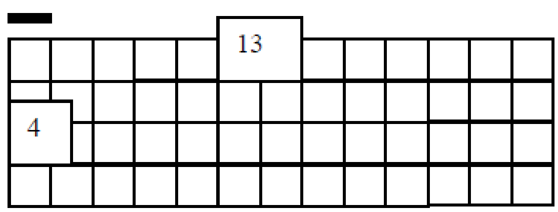
Rectangle	Length (in.)	Width (in.)	Area (in.^{2})	Perimeter (in.)
A	5	6	■	■
B	4	13	■	■
C	$6\frac{1}{2}$	8	■	■

Rectangle	Length (in.)	Width (in.)	Area (square in.)	Perimeter (in.)
A	5	6	30	22
B	4	13	52	34
C	$6\frac{1}{2}$	8	52	29

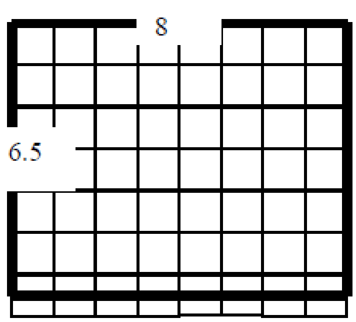
Possible Sketches:



A



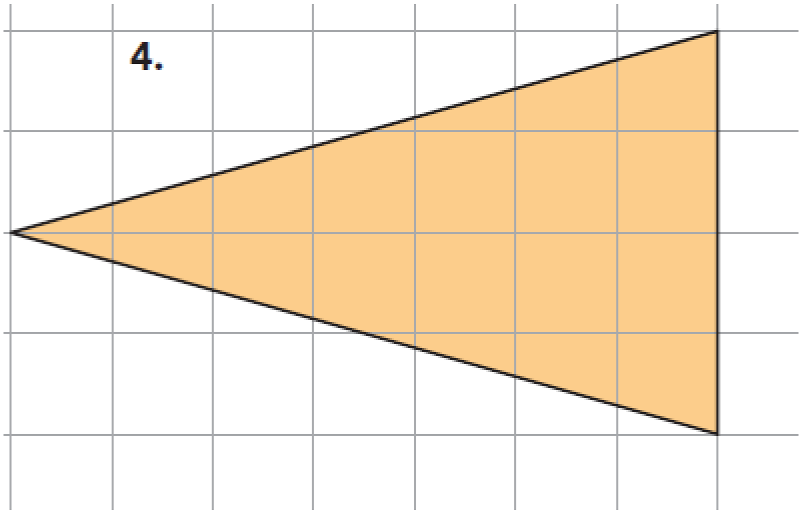
B



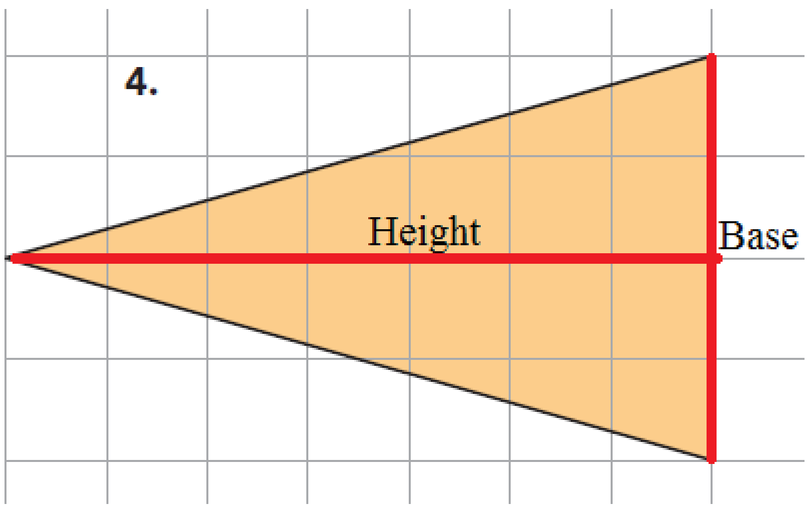
C

Investigation 2: *Measuring Triangles*
ACE #4

Calculate the area and perimeter of the triangle and explain your reasoning.



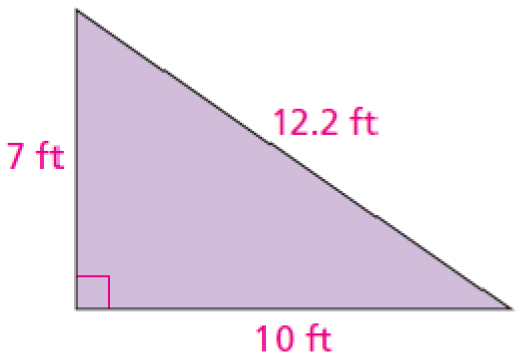
Students have several ways to think about the area of the triangle. They might count whole square units covered and then estimate the area covered by the partial squares. Or they might surround this with a 4 by 7 rectangle and observe that the triangle is half of the rectangle. Or they might use the rule that Area of triangle = (1/2)(base)(height) and use the base and height shown on the picture below.



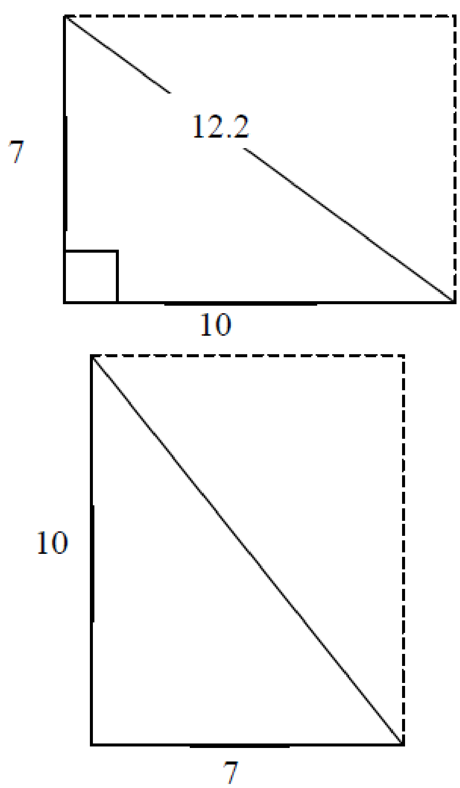
Area = 0.5 (4)(7) = 14 square units
Perimeter = 4 + length of two other sides of the triangle.
The problem we have with the lengths of two other sides of the triangle is that they do not lie on grid lines, so have to be measured or estimated using the edge of a grid square as a unit. Each is approximately 7.3 units long. (In a later unit students learn how to use the Pythagorean Theorem to find an accurate answer for these lengths).
Perimeter ≈ 4 + 7.3 + 7.3 ≈ 18.6 units.

Investigation 2: *Measuring Triangles*
ACE #9

Vashon said that if you used 7 feet as the base of the triangle shown below then you would calculate the same area as you did when you used the10 feet base. Do you agree with him?



Vashon is correct. It does not matter which side of a triangle we choose as the base, as long as we then choose as the height the distance from the base to the opposite vertex. The triangle is half of the same 7 by 10 rectangle no matter the orientation.



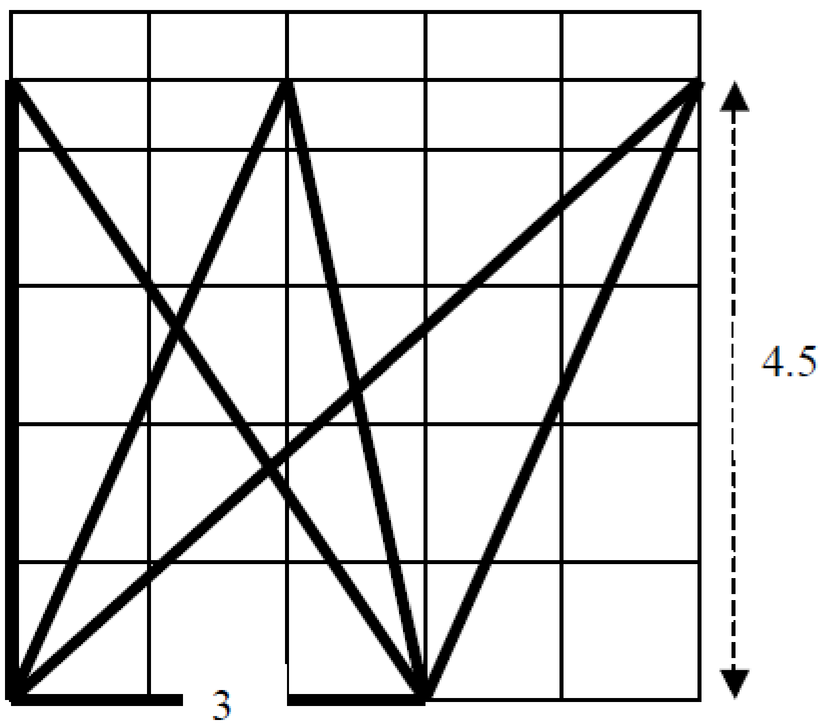
Investigation 2: *Measuring Triangles*
ACE #12

Melissa was finding the area of a triangle when she wrote:

$$\text{area} = \frac{1}{2}\left(3 \times 4\frac{1}{2}\right)$$

a. Make a sketch of a triangle she might have been working with.
b. What is the area of the triangle?

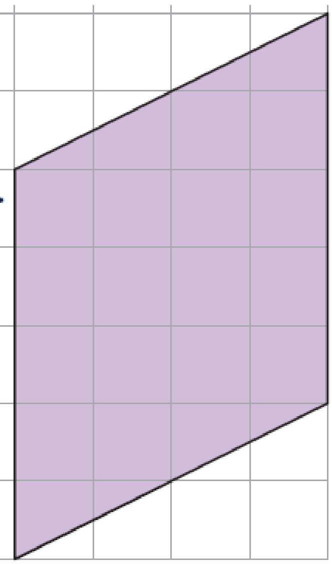
Apparently Melissa is using a base of 3 and a height of 4.5 for her triangle. But there are many triangles she might be working with. The key is to make the height be the perpendicular distance from the base to the opposite vertex. Shown below are several different triangles with the same base and height (and, therefore, the same area.)



Investigation 3: *Measuring Parallelograms*
ACE #7

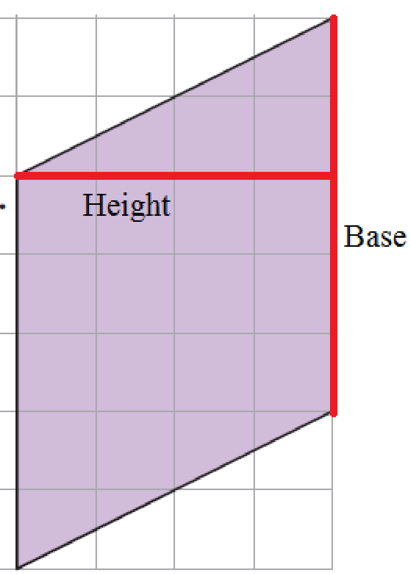
For exercises 1-7, find the area and perimeter of each parallelogram. Explain how you found your answers for parallelograms 2, 6 and 7.

7.



Area = 20 cm^2 (base = 5 cm, height = 4 cm), perimeter ≈ 19 cm (The side lengths are 5 cm and about 4.5 cm).
Students have several ways to think about the area of the parallelogram. They might count whole square units covered and then estimate the area covered by the partial squares. Or they might partition shape into triangles and rectangles and complete the shape into a 4 by 5 rectangle by moving one of the triangles. Or they might use the rule that Area of parallelogram = (base)(height) and use the base and height shown on the picture below.

7.

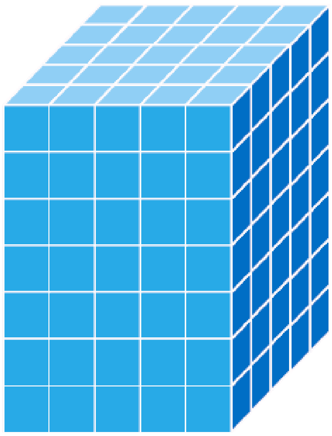


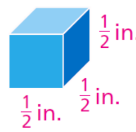
Investigation 4: *Measuring Surface Area and Volume*
ACE #17

For Exercises 15–17, a rectangular prism is built with 1/2-inch cubic blocks.

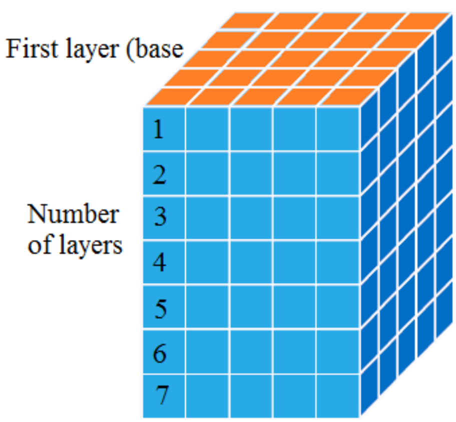
- Find the length, width, and height of each prism.
- How many 1/2-inch cubes are needed to fill the box?
- What is the volume of each prism in cubic inches?
- Find the surface area of each prism.

17.





There are 5 ½-inch cubic blocks over the length. Length = 2.5 in.
There are 5 ½-inch cubic blocks over the width. Width = 2.5 in.
There are 7 ½-inch cubic block over the height. Height = 3.5 in.
To find the number of ½-inch cubic units needed to fill the box we can find the number of cubes in the first layer, we call it as base layer and multiply that with number of layers (see the figure below).
There are 25 cubes in the base layer and we have 7 layers so we need 175 ½-inch cubes to fill the box.



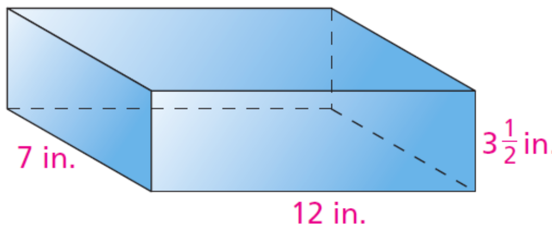
Or you can multiply length, width and height.
Volume = 5 ½-inch cubes x 5 ½-inch cubes x 7 ½-inch cubes = 175 cubic ½-inch cubes.
Volume in cubic inches = 2.5 in x 2.5 in x 3.5 in = 21.875 cubic inches.

Surface area is the sum of all unit squares that fit on the exterior of this rectangular prim.
Surface area = 41 1/2 in^2.

Investigation 4: *Measuring Surface Area and Volume*
ACE #22

For exercises 20-23, find the volume and surface area of each rectangular prism.

22.



Volume = 7 in x 12 in x 3.5 in = 294 in^3

Surface area is the sum of all unit squares that fit on the exterior of a solid. To find the surface area of a rectangular prism, we need the dimensions, or the length, width, and height, of the prism. Rectangular prisms have six surfaces: a front, back, top, bottom, left, and right. The surface area of a prism is the sum of all the areas of these rectangles. The top and bottom faces will have equal areas, so will the front and back faces, and the left and right faces. Instead of finding the areas of all six faces, we can find the areas of three unique faces, multiply each by 2, and then find their sum.
Surface Area = 2 (7 by 12 rectangles) + 2 (7 by 3.5 rectangles) + 2 (12 by 3.5 rectangles)
Surface Area = 2x(7x12) + 2x(7x3.5) + 2x(12x3.5)
Surface Area = 301 square inches.