Variables and Patterns: Homework Examples from ACE

Investigation 1: *Variables, Tables, and Graphs* ACE #7
Investigation 2: A*nalyzing Relationships among Variables,* ACE #17
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Investigation 1: *Variables, Tables, and Graphs*
ACE #7

Below is a chart of the water depth in a harbor during a typical 24-hour day. The water level rises and falls with the tides.

Effect of the Tide on Water Depth

Hours Since Midnight	0	1	2	3	4	5	6	7	8
Depth (m)	10.1	10.6	11.5	13.2	14.5	15.5	16.2	15.4	14.6

Hours Since Midnight	9	10	11	12	13	14	15	16
Depth (m)	12.9	11.4	10.3	10.0	10.4	11.4	13.1	14.5

Hours Since Midnight	17	18	19	20	21	22	23	24
Depth (m)	15.4	16.0	15.6	14.3	13.0	11.6	10.7	10.2

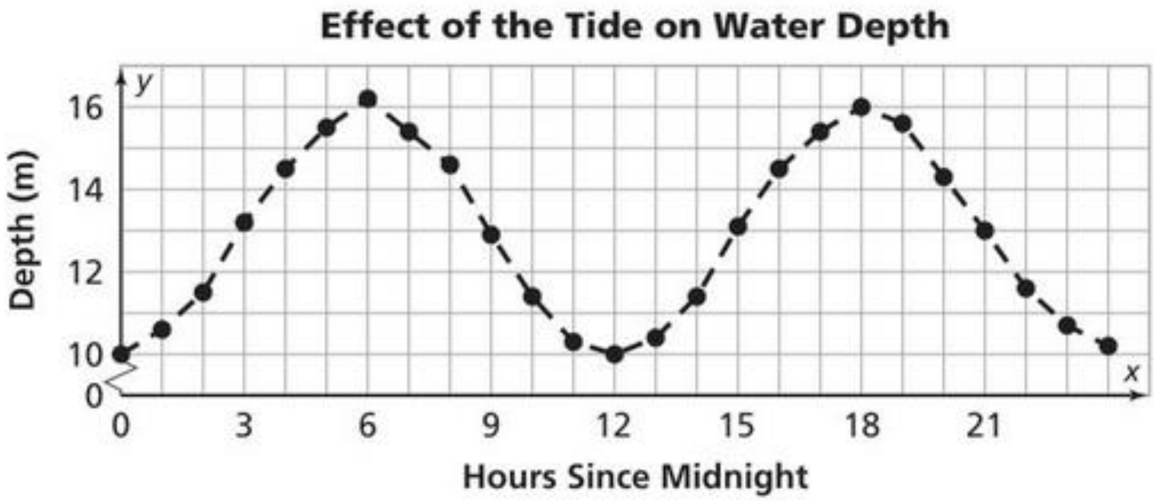
a. At what time is the water the deepest? Find the depth at that time.
b. At what time is the water the shallowest? Find the depth at that time.
c. During what time interval does the depth change most rapidly?
d. Make a coordinate graph of the data. Describe the overall pattern you see.
e. How did you choose scales for the x-axis and y-axis of your graph? Do you think everyone in your class used the same scales? Explain.

a. The water is deepest at 6 hours after midnight, or 6:00 a.m., with a depth of 16.2 m.

b. The water is shallowest at noon with a depth of 10.0 m.

c. Water depth changes most rapidly (by 1.7 meters per hour) between 2 a.m. and 3 a.m., between 8 a.m. and 9 a.m., between 2 p.m. and 3 p.m. This pattern shows the physical property of tides that they move most swiftly at points halfway between high and low tides (which occur roughly every six hours).

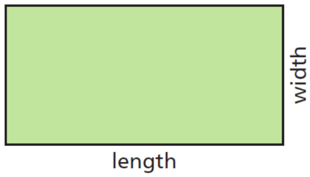
d. The pattern of the graph is bimodal (two humps). It looks symmetric, so that if it was flipped over when x = 12 (hour 12), the two parts would line up. Overall, the graph rises to hour 6, then the water depth goes back down, and then rises again to hour 18, and then the depth decreases again. See graph below.



e. Possible answer: I used 1-hour intervals on the x-axis because these were the time intervals given in the table. I used 2-meter intervals on the y-axis because it allowed all the data to be graphed on my grid paper. (Not all students will use this scale. They might use 1 meter intervals on the vertical axis, because the numbers range from 10 to 16.2,not a large range. Or they might want to use 0.5 meter intervals or even smaller, trying to show the decimal numbers more accurately. It depends on how much room they have vertically. They do not have to show the numbers 0 – 9 on the vertical axis since these are not used, but if they omit these then they must indicate that this has been done, as above. They should not simply mark 0 then 10 on this axis. Above all, increments on the axes must have the same values, with tick marks every 1 or every 2 or every 0.5 meter, for example. A common error is to mark the vertical axis with the numbers given in the table.

Investigation 2: A*nalyzing Relationships among Variables*
ACE #17 (from connections)

The area of a rectangle is the product of its length and its width.



a. Find all whole-number pairs of length and width values that give an area of 24 square meters. Copy and extend the table here to record the pairs.

Rectangles With Area 24 m^2

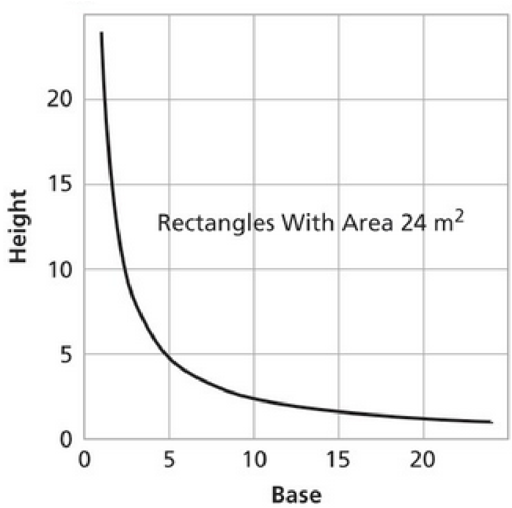
Length	■	■	■	...
Width	■	■	■	...

b. Make a coordinate graph of the (length, width) data from part (a).
c. Connect the points on your graph if it makes sense to do so. Explain your decision.
d. Describe the relationship between length and width for rectangles of area 24 square meters.

a. (Notice the connection here with factors from Prime Time)

Length	1	2	3	4	6	8	12	24
Width	24	12	8	6	4	3	2	1

b.



c. It makes sense to connect the points because rectangles can have any length and width with product 24 (e.g., 1.5 and 16).

d. Possible answer: As the length increases the width decreases, rapidly when length is small, and then more slowly as length gets larger.

Investigation 3: *Relating Variables with Equations*
ACE #14

The sales tax in a state is 8 %. Write an equation for the amount of tax *t* on an item that costs *d* dollars.

Say we made a purchase of $1.00 then the tax is $0.08, for $2.00 the tax is $0.16 etc. In a table this is

Cost *d* $	1	2	3	4
Tax *t* $	0.08	0.16	0.24	0.32

Our equation will be $t = 0.08d$.

Investigation 3: *Relating Variables with Equations*
ACE #15

Potatoes sell for $ .25 per pound at the produce market. Write an equation for the cost c of p pounds of potatoes.

$c = 0.25p$

Investigation 3: *Relating Variables with Equations*
ACE #16

A cellphone family plan costs $49 per month plus $.05 per text. Write an equation for the monthly bill b when t texts are sent.

Say we talk for 1 minute then the Bill is $49 + $0.05, for 2 minutes, $49 + $0.10 etc. In a table this is

Minutes, m	1	2	3	4
Bill, $b	49.05	49.10	49.15	49.20

Students may find the two bits of information distracting and want to try $B = 49m$ or $B = 0.05m$, neither of which produces the pairs in the table. To get the pairs in the table we hold the $49 constant, no matter how many minutes and change the amount added as m changes.

$b = 0.05m + 49$

Investigation 3: *Relating Variables with Equations*
ACE #17, #18, #19

For Exercises 17–19, describe the relationship between the variables in words and with an equation.

17.

x	0	1	2	5	10	20
y	0	4	8	20	40	80

18.

s	0	1	2	3	6	12
t	50	49	48	47	44	38

19.

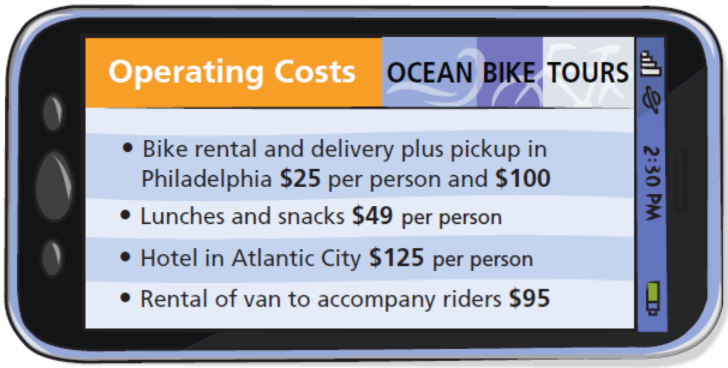
n	0	1	2	3	4	5
z	1	6	11	16	21	26

17. Students will observe that the values of y increase by a constant rate of 4 for each increase of 1 in x. y = 4x.

18. Students will notice that the t values decrease by 1 as s increases by 1. They may try t = 49s, if they only look at the first pair. They may try 49s – 1 or 49 – s or other variations, as they try to think out how “49” and “-1” combine to produce these pairs. If the y-intercept were given (0,50) this would be an additional clue that helps. t = 50 - s.

19. Students will observe that the values of z increase by a constant rate of 5 for each increase of 1 in n. Again, if the y-intercept is given (or worked out, by working backwards) then the pair (0, 1) would be an additional clue. z = 5n +1

Investigation 4: *Expressions, Equations, and Inequalities*
ACE #8



a. Write a rule that shows how total operating cost C depends on the number n of riders. The rule should show how each cost variable adds to the total.
b. Write another rule for total operating cost C. This rule should be as simple as possible for calculating the total cost.
c. Give evidence showing that your two expressions for total cost are equivalent.

a. $C = (100+25n) + 49n + 125n + 95$
b. $C = 199n + 195$
c. Students might come up with different explanations. Three possible explanations:
Explanation 1: Tables of sample cost values are identical, as are graphs of two relationships.
Explanation 2: Each person's separate costs (25, 49, and 125) are equal to a total of 199 per person, and the 100 and 95 are fixed independent of number of customers.
Explanation 3: use the Distributive Property to find $25n + 49n = (25+49)n = 74n$. Again use the Distributive Property to find $74n + 125n = (74+125)n = 199n$. Then the expression $100+199n+95$ is equal to $199n+100+95$ because of the Commutative Property, and that expression simplifies to $199n+195$.

Investigation 4: *Expressions, Equations, and Inequalities*
ACE #15

A baseball team wanted to rent a small bus for travel to a tournament. Superior Bus charges $ 2.95 per mile driven. Coast Transport charges $300 plus $2 per mile. Use these data for Exercises 13–16.

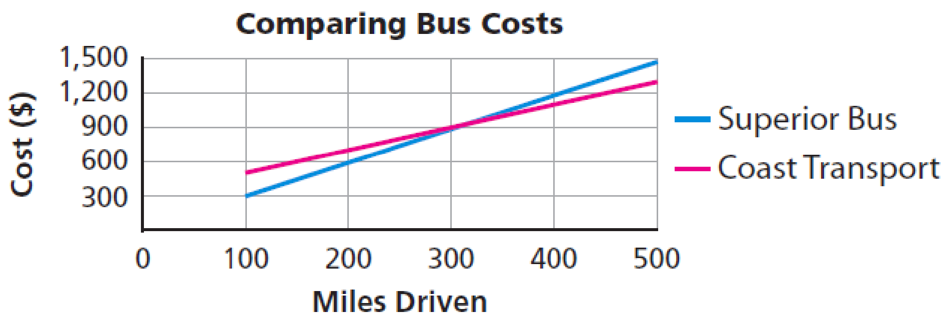
15. The rental for a bus from Coast Transport was $600.
• Write and solve an equation to find the distance driven.
• Check your solution by substituting its value for the variable m in the equation.
• Explain how you found the solution.

The equation for the rental cost of $600 for a bus from Coast Transport is $600 = 300 + 2m$.
The solution for $600 = 300 + 2m$ is $m = 150$.
Check: $300 + 2(150) = 600$.
Possible explanations:
Inspect a table or graph of the equation $R = 300 + 3m$, looking for values of the independent variable m (miles) that produce a value of 600 for the dependent variable R (rental cost).
Or Use inverse operations –subtract 300 from 600 and then divide by 2.

Investigation 4: *Expressions, Equations, and Inequalities*
ACE #16

The team wanted to know which bus company's offer was a better value. Use the table and graph below to answer their questions.

Miles	Superior Bus	Coast Transport
100	295	500
200	590	700
300	885	900
400	1180	1100
500	1475	1300



There are several ways to estimate solutions for equations. The simplest method is often called guess and check. It involves three basic steps.

- Make a guess about the solution.
- Check to see if that guess solves the equation.
- If it does not, revise your guess and check again in the equation.

a. The two rental companies charge the same amount for one distance. Write an equation to find that distance. Then solve the equation by guess and check. (The table and graph will help.)
b. For what numbers of miles will the charge by Superior Bus be less than that by Coast Transport?
c. For what numbers of miles will the charge by Coast Transport be less than that by Superior Buses?

a. Equation is 2.95 m = 300 + 2m. The table shows charges are equal for about 300 miles, so that is the first guess. Exact distance is 316.
b. The charge by Superior Bus will be less than that by Coast Transport for travel of less than about 300 miles (precisely 315 miles). We can use table or graph to guide our estimate. In the graph until 300 miles the blue line representing Superior Bus is below the red line representing Coast Transport.
c. The charge by Coast Transport will be less than that by Superior Bus for travel of more than 300 miles (precisely 316 miles).