Accentuate the Negative: Homework Examples from ACE

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Investigation 1: *Extending the Number System*
ACE #6, 7

For Exercises 6 and 7, find each Math Fever team's score. Write a number sentence for each team. Assume that each team starts with 0 points.

6. **Protons**

Point Value	Answer
250	Correct
100	Correct
200	Correct
150	Incorrect
200	Incorrect

7. **Neutrons**

Point Value	Answer
200	Incorrect
50	Correct
250	Correct
150	Incorrect
50	Incorrect

6. 250 + 100 + 200 + (-150) + (-200) = 200 or 250 + 100 + 200 – 150 – 200 = 200

7. (-200) + 50 + 250 + (-150) + (-50) = -100 or -200 + 50 + 250 – 150 – 50 = -100

This game context was used for in-class problems to introduce students to positives and negatives and combining these quantities.

Investigation 1: *Extending the Number System*
ACE #12-15

Copy each pair of numbers in Exercises 12-15. Then insert <, >, or = to make each a true statement.

12. 3 ? 0

13. -23.4 ? 23.4

14. 46 ? -79

15. -75 ? -90

12 . 3 is greater than 0, or 3 > 0.

13 . -23.4 is less than 23.4, or -23.4< 23.4 1

14. 46 > -79

15. -75 > -90

Investigation 1: *Extending the Number System*
ACE #47

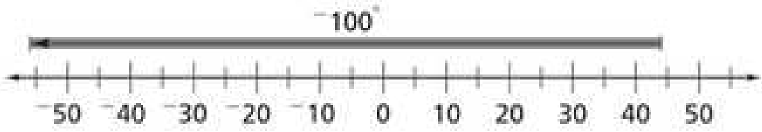
The greatest recorded one-day temperature change occurred in Browning, Montana (bordering Glacier National Park), from January 23-24, 1916. The temperature fell from 44 degrees to -56 degrees in less than 24 hours.

a. What was the temperature change that day?
b. Write a number sentence to represent the change.
c. Show the temperature change on a number line.

a. There are multiple ways to think about this problem. It takes 44 units to drop from 44 to 0, and then another 56 units to drop from 0 to -56. This is a total change (drop) of 44 + 56 = 100 degrees! If students think of this as a direction as well as a change, they are thinking of the difference from 44 to -56, that is, -56 – 44 = -100, *down* 100 degrees.

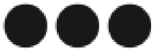
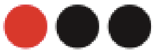
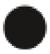
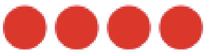
b. Two possible answers: -56 – 44 = -100, or 44 + -100 = -56.

c.



Investigation 1: *Extending the Number System*
ACE #49-52

For Exercises 49–52, find the missing part for each chip problem. Write a number sentence for each problem.

	Start With	Rule	End With	Number Sentence
49.		Add 5 	■	■
50.		Subtract 3 	■	■
51.		■		■
52.	■	Subtract 3 		■

In the chip board model, the black circles ("Bs") stand for a black chip with a value of positive 1, and the red circles ("Rs") stand for a red chip with value negative 1. So, 1B + 1R = 0, 3B + 3R = 0 etc.

49. Start with +3 and add -5. We can think of each pair of "B + R" as 1 + (-1) = 0. Since there are 2 more red chips we end with two red chips for a value of -2: +3 + -5 = -2.

50. Start with -1 + 2 and subtract (-3). Since there are not enough reds to subtract three reds, we could alter the original representation from -1 + 2 to, for example, -3 + 4 by adding two more red chips *and* two more black chips. Notice that this change does not actually change the value of the board, but it does allow us to take away three reds (subtract -3). So now we have three red chips and 4 black chips, and when we take away three red chips, the end result is four black chips for a value of +4: -1 + 2 – (-3) = 4.

Notice that after the "3R" has been subtracted, the net result is the addition of 3B to the board. This helps explain why subtracting -3 is the same as adding +3.

51. Start with -5 and "do something" so that we end with -2. This could be -5 add 3, or "add three blacks chips." Or students might think of this as-5 – (-3), which would be "take away three reds." The number sentence would be: -5 – (-3) = -2 or -5 + 3 = -2.

Notice that again, we see that adding +3 gives the same result as subtracting -3.

52. There are several possible answers, but here's the general idea: We are starting with some unknown value, then subtracting positive 3 (taking away three black chips) to end with -4. We must have started with -4 + 3 = -1 or some combination of chips that makes a value of -1 (such as one red chip, two black chips and 3 reds, four reds and three blacks, and so on). A possible number sentence would be: -1 – 3 = -4.

Investigation 2: *Adding and Subtracting Rational Numbers*
ACE #18-22

Use your algorithms to find each difference without using a calculator. Show your work.

18. 12 – 4 19. 12 – 12 20. –12 – 12

21. –7 – 8 22. 45 – (-40)

An *algorithm* is an efficient and logical procedure to solve a problem. Some students' algorithm may involve using the chip model or a number line. For some students, the algorithm for subtraction is a rule that they have observed always works. For example, for these problems, to subtract an integer we can add the opposite. See notes above for Investigation 1 for more on subtracting integers.

18. Students can think of this as a chip board model (12 black chips take away 4 blacks) or as a number line model ("what is the difference from 4 to 12?" Or, "start at 12 on the line and come down 4 units). Or they may rewrite this as an addition: +12 – (+4) = +12 + (-4) = 8.

19. Students can think of this as a chip board model (12 blacks take away 12 blacks) or as a number line model ("what is the difference from 12 to 12 on the line?" Or, "start at 12 on the line and go down 12 units"). Or they may rewrite this as an addition: 12 – 12 = 12 + (-12) = 0.

20. Again, students can think of this as a chip board model (12 red chips take away 12 black chips, which requires us to add 12 of each kind of chip so we maintain the value of the board but have enough black chips to remove) or as a number line model ("what is the difference from -12 to 12 on the line?" Or, "start at -12 on the line and go down 12 units"). Or they may rewrite this as an addition: -12 – 12 = -12 + (-12) = -24.

21. -7 – 8 = -7 + (-8) = -15

22. 45 – (-40) = 45 + 40 = 85

Investigation 2: *Adding and Subtracting Rational Numbers*
ACE #38(a)

Without doing any calculations, decide which expression is greater. Explain your reasoning.

a. 5,280 + (-768) OR 5,280 – (-768)

Both expressions start with +5280, but one adds a negative and the other subtracts a negative. The first expression will be *less* than +5280. If we think in terms of the chip board model then the second computation, which involves subtracting a negative, would require a re-representation of the initial value +5280 by adding the 768 "positives" and "768" negatives, before taking away the negatives. This ends with a *larger* result than +5280. So the second expression is greater than the first.

Investigation 2: *Adding and Subtracting Rational Numbers*
ACE #48 (a) – (d), i

Compute each of the following.

a. 3 + -3 + -7 b. 3 – 3 – 7 c. -10 + -7 + -28 d. -10 – 7 – 28

i. What can you conclude about the relationship between subtracting a positive number and adding a negative number with the same absolute value? In other words, what is the relationship between a (– $^{+}$) situation and a (+ $^{-}$) situation?

a. 3 + (-3) + (-7) = 0 + (-7) = -7

b. 3 – (+3) – (+7) = 0 –(+7) = -7

c. -10 + (-7) + (-28) = (-17) + (-28) = -45

d. -10 – (+7) –(+28) = -10 – (35) = -45

i. Based on parts (a) and (b), It seems that "add (-3)" gives the same result as "subtract (+3)". In general, this suggests that "add -A" gives the same result as "subtract +A".

Note: This rule generalizes to be "Adding any integer gives the same result as subtracting its opposite, or subtracting any integer gives the same result as adding its opposite."

Investigation 2: *Adding and Subtracting Rational Numbers*
ACE #56

For each of the following, write a related equation. Then find the value of *n*.

a. $n - 7 = 10$ b. $-\frac{1}{2} + n = -\frac{5}{8}$ c. $\frac{2}{3} - n = -\frac{7}{9}$

a. $n = 10 + 7$. So $n = 17$.

b. $n = -\frac{5}{8} + \frac{1}{2}$. So $n = -\frac{1}{8}$.

c. $n = \frac{7}{9} + \frac{2}{3}$. So $n = \frac{13}{9}$

Note: In elementary school, most students learned "fact families" for any addition or subtraction. The idea is that for addition and subtraction facts, there are multiple ways to represent the same relationship. For example, 3 + 4 = 7 shows the same relationship as 7 – 4 = 3 and 7 – 3 = 4.

Investigation 3: *Multiplying and Dividing Rational Numbers*
ACE #14

You have located fractions such as $-\frac{5}{7}$ on a number line. You have also used fractions to show division: $\frac{-5}{7}$ = -5 ÷ 7 and $\frac{5}{-7}$ = 5 ÷ (-7). Tell whether each statement is *true* or *false*. Explain.

a. $\frac{-1}{2} = \frac{1}{-2}$ b. $-\frac{1}{2} = \frac{-1}{-2}$

a. True. You can either distribute the negative sign (that is out in front of the fraction—think of this as $-\frac{1}{2}$) to the numerator OR the denominator. In either of the forms, it will still be a negative number.

b. False. You can think of it as $-\frac{1}{2}$ = -0.5, but $-\frac{-1}{-2}$ = 0.5. In $\frac{-1}{-2}$, both numbers are negative, and a negative divided by a negative equals a positive.

Investigation 3: *Multiplying and Dividing Rational Numbers*
ACE #38

The Extraterrestrials have a score of -300. They answer four 50-point questions incorrectly. What is their new score?

The answer is -500 points; -300 +4(-50) = -300 + (-200) = -500, or -300 – (4 · 50) = -500.

Investigation 4: *Properties of Operations*
ACE #8-11

For Exercises 8-11, rewrite each expression in an equivalent form to show a simpler way to do the arithmetic. Explain how you know the two results are equal without doing any calculations.

8. (-150 + 270) + 30
9. (43 · 120) + [43 · (-20)]
10. 23 + (-75) + 14 + (-23) – (-75)
11. [0.8 · (-23)] + [0.8 · (-7)]

8. Since all the operations are addition, we can change the grouping (Associative Property of addition) and order (Commutative Property) of the numbers. So one possible answer is (-150 + 270) + 30 = -150 + (270 + 30). This has the advantage of putting the positive numbers together and also of creating a “friendly” pair of numbers to add, since 270 + 30 = 300. This gives -150 + (300) = 150.

9. There are two expressions added here, and each has a common factor of 43. So we can use the Distributive Property to rewrite this as 43(120 + -20), which is 43(100) = 4300.

10. This expression has addition and subtraction, and can be rewritten in terms of additions only. Thus, 23 + -75 + 14 + -23 – (-75) = 23 + -75 + 14 + -23 + 75, and then the order can be changed (since addition is commutative) to 23 + -23 + -75 + 75 +14. Then, taking advantage of opposites, we have a final result of 14.

11. The Distributive Property can be used to factor 0.8 out of both expressions. Then, we have 0.8(-23 + -7) = 0.8(-30) = -24.

Investigation 4: *Properties of Operations*
ACE #45 (a)-(d)

Insert parentheses (or brackets) in each expression if needed to make the equation true.

a. 1 + (-3) · (-4) = 8

b. 1 + (-3) · (-4) = 13

c. -6 ÷ (-2) + (-4) = 1

d. -6 ÷ (-2) + (-4) = -1

This problem asks students to apply the parentheses so that the correct order of operations will give the required result. The order of operations is: Operations in *parentheses* (or brackets) first, then *exponents*, then *multiplication or division from the left*, then *addition or subtraction from the left*.

a. (1 + - 3) · -4 = (-2) · (-4) = 8

b. 1 + (-3 · -4) = 1 + 12 = 13

c. -6 ÷ (-2 + -4) = -6 ÷ -6 = 1

d. (-6 ÷ -2) + -4 = 3 + -4 = -1