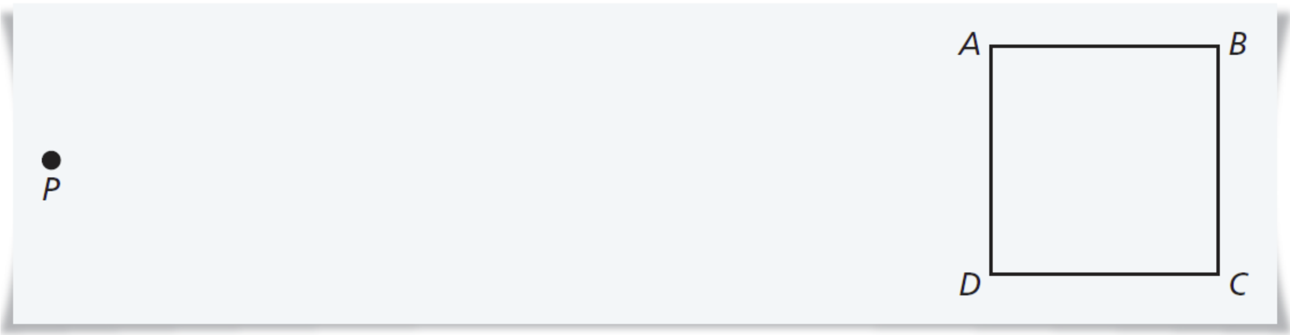
Stretching and Shrinking: Homework Examples from ACE

Investigation 1: *Enlarging and Reducing Shapes* ACE #3
Investigation 2: *Similar Figures,* ACE #3, #10
Investigation 3: *Scaling Perimeter and Area,* ACE #19, #20, #25
Investigation 4: *Similarity and Ratios,* ACE #3, #11

Investigation 1: *Enlarging and Reducing Shapes*
ACE #3

Copy square *ABCD* and anchor point *P* onto a sheet of paper. Use the rubber-band method to enlarge the figure. Then, answer parts (a)–(e) below.



a. How do the side lengths of the original figure compare to the side lengths of the image?
b. How does the perimeter of the original figure compare to the perimeter of the image?
c. How do the angle measures of the original figure compare to the angle measures of the image?
d. How does the area of the original figure compare to the area of the image? How many copies of the original figure would it take to cover the image?
e. How does the distance between each point in the original figure and P compare to the corresponding distances in the image?

a. The side lengths should be twice as long. (This is because we used a 2 band stretcher. If we used a 3 band stretcher the image lengths would be three times as long. Students may not realize that it is because the distances from the *image to the anchor point* are all twice as long as the corresponding distances from the *original to the anchor point* that the image is twice as large. There are, in fact, similar triangles embedded in the drawing.)

b. The perimeter should also be twice the original. (This makes sense since each length has been doubled, so the sum of all the sides will also have been doubled.)

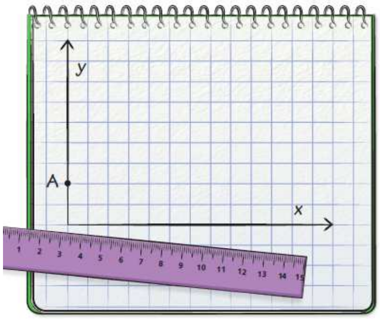
c. The angles are still the same, 90 degrees.

d. The area has been increased by a factor of 4. (This is often a surprise to students who think that if the lengths are doubled the area should be doubled also. You can convince a student of this by drawing a square with sides of 1 inch, and another square with sides of 2 inches. Four of the smaller squares fit inside of the larger square.)

e. The distance of each image point is twice the distance from P as its corresponding point on the original figure.

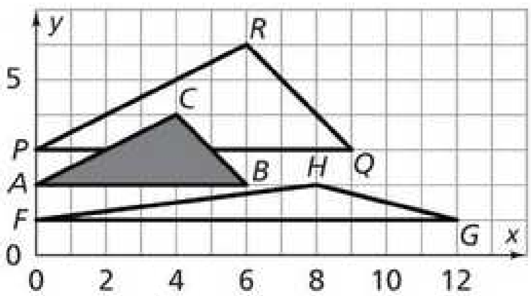
Investigation 2: *Similar Figures*
ACE #3

a. On grid paper, draw triangle ABC with vertex coordinates A(0, 2), B(6, 2), and C(4, 4).



b. Apply the rule (1.5x, 1.5y) to the vertices of triangle ABC to get triangle PQR. Compare the corresponding measurements (side lengths, perimeters, areas, area, angle measures) of the two triangles.
c. Apply the rule (2x, 0.5y) to the vertices of triangle ABC to get triangle FGH. Compare the corresponding measurements (side lengths, perimeters, areas, angle measures) of the two triangles.
d. Which triangle, PQR or FGH, seems similar to triangle ABC? Why?

Figure for **a, b and c**



b. (0,2) becomes (0, 3). (6,2) becomes (9, 3). (4, 4) becomes (6, 6). The sides of the image are 1.5 times as long as the original. The perimeter is 1.5 times as long. The area of the original triangle, using the formula 0.5(base)(height), is (0.5)(6)(2) = 6 square units. The area of the image triangle is (0.5)(9)(3) = 13.5. So the new area is 2.25 times the original. The angle measures have not changed.
c. (0, 2) becomes (0, 1). (4, 4) becomes (8, 2), (6, 2) becomes (12, 1). The length of FG is twice the length of AB. The length of FH is more than, but not twice, AC. (Actually if you apply the Pythagorean theorem we can find the lengths, but students have not studied this yet, so they will have to rely on estimation.). Likewise the length of GH is more than but not twice BC. The area of triangle ABC is 6 square units. The area of triangle FGH is (0.5)(12)(1) = 6 square units. (This makes sense. We doubled one dimension and halved the other so the area remains unchanged.) The angles are quite changed.
d. Angles in triangle PQR are equal to corresponding angles in ABC, and lengths in PQR are double the corresponding lengths in ABC. So triangle PQR is similar to triangle ABC. Note: similar figures have the same shape (corresponding angles must be the same size), and are scale copies of each other (corresponding sides must be in the same ratio).

Investigation 2: *Similar Figures*
ACE #10

What is the scale factor from an original figure to its image if the image is made using the given method?
a. a two-rubber-band stretcher
b. a copy machine with size factor 150,
c. a copy machine with size factor 250,
d. the coordinate rule (0.75x, 0.75y)

a. The scale factor will be 2. (Ratio of length in image: corresponding length in original = 2:1. Also, distance from a point on the image to the anchor point is twice the distance of the corresponding point on the original to the anchor point.)

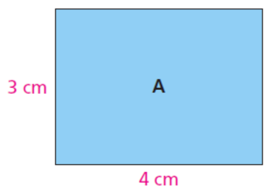
b. 1.5 (Ratio of length in image: corresponding length in original = 150:100 or 1.5:1)

c. 2.5

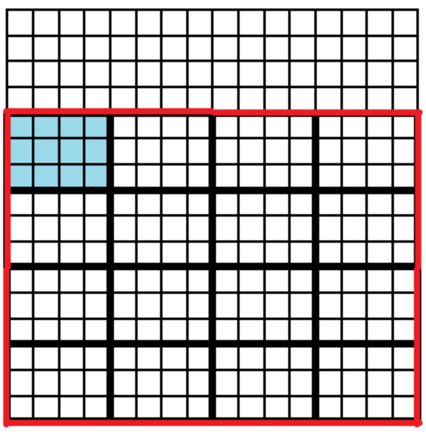
d. 0.75 (Ratio of length in image: corresponding length in original = 0.75:1)

Investigation 3: *Scaling Perimeter and Area*
ACE #19

Suppose Rectangle B is similar to Rectangle A below. The scale factor from Rectangle A to Rectangle B is 4. What is the area of Rectangle B?



A scale factor of 4 (from A to B) means that lengths of rectangle A have to be multiplied by 4 to make the lengths of rectangle B. Thus, the sides of rectangle B are 3(4) cm and 4(4) cm. This means that the area of rectangle B is 12 x 16 square centimeters or 192 square centimeters. Note: Rectangle B is 16 times as large as Rectangle A.
(In investigation 3 students combined multiple copies of specific shapes to create a larger, similar shape. The original shapes were named *rep-tiles*. In this case, rectangle A is a *rep-tile* and 16 copies of the rectangle would be needed to create rectangle B.)



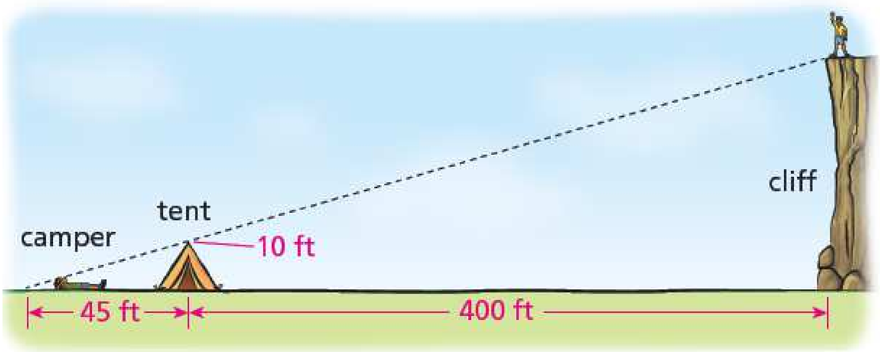
Investigation 3: *Scaling Perimeter and Area*
ACE #20

Suppose Rectangle E has an area of 9 square centimeters and Rectangle F has an area of 900 square centimeters. The two rectangles are similar. What is the scale factor from Rectangle E to Rectangle F?

The area of Rectangle F compared to the area of Rectangle E is 900 ÷ 9 = 100.
The scale factor *squared* tells us how many times greater (or less) the area of the image is compared to the area of the original figure. If scale factor times scale factor is 100, scale factor is 1/10. The scale factor from Rectangle E to Rectangle F is 10.

Investigation 3: *Scaling Perimeter and Area*
ACE #25

Judy lies on the ground 45 feet from her tent. Both the top of the tent and the top of a tall cliff are in her line of sight. Her tent is 10 feet tall. About how high is the cliff? Assume the two triangles are similar.



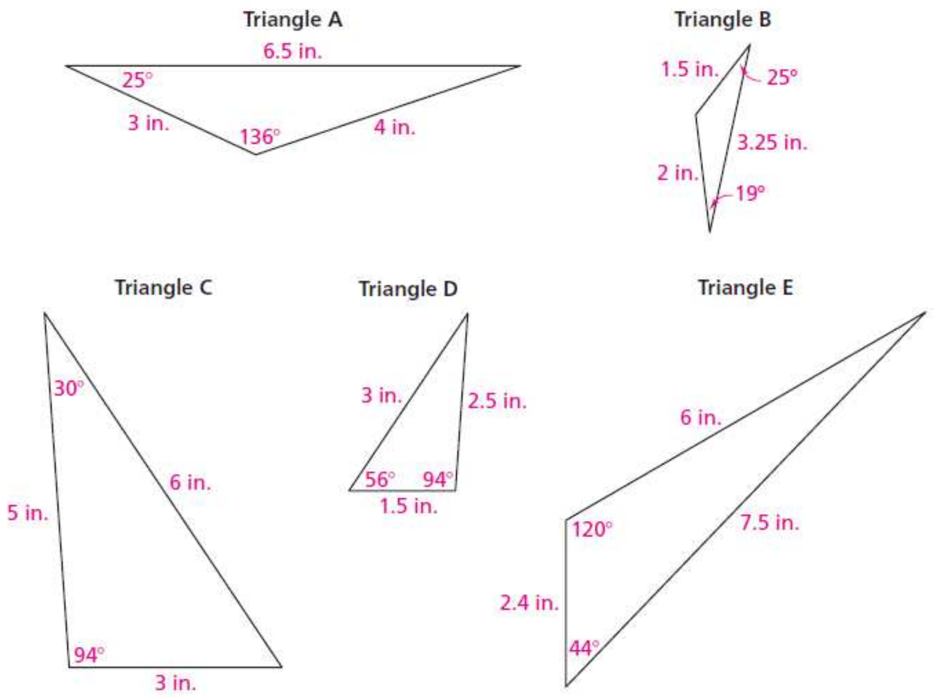
Not drawn to scale

The scale factor from small to large is 445/45. Therefore, to find the height of the cliff we need to multiply 10 by the same scale factor. The cliff height is 10(445/45) = 98.9 feet approximately. (Alternative solution strategies might involve setting up equal ratios: cliff/10 = 445/45 etc.)

Note: we only need to know that corresponding angles in two triangles are equal to deduce that the triangles are similar. We can then figure out missing lengths. BUT, knowing that corresponding angles are equal is NOT enough information to show that other figures are similar.

Investigation 4: *Similarity and Ratios*
ACE #3

For parts (a)–(d), use the triangles below. The drawings are not to scale.



a. List all the pairs of similar triangles. Explain why they are similar.
b. For each pair of similar triangles, find the ratio of two side lengths in one triangle. Find the ratio of the corresponding side lengths in the other. How do these ratios compare?
c. For each pair of similar triangles, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different than the information given by the ratios of side lengths.
d. How are corresponding angles related in similar triangles? Is it the same relationship as for corresponding side lengths? Explain.

a. A and B are similar. C and D are similar. They are similar because their corresponding angle measures are congruent. Also, each ratio of adjacent side lengths within one figure is equivalent to the ratio of corresponding side lengths in the similar figure. Lastly, scale factors from each side length in one figure to the corresponding side length in the similar figure are constant.

b. The side-length ratios for Triangle A are 3 to 4, 3 to 6.5, and 4 to 6.5.

The corresponding side-length ratios for Triangle B are 1.5 to 2, 1.5 to 3.25, and 2 to 3.25.

When simplified, the ratios for Triangle A and Triangle B are equivalent:
$\frac{3}{4} = \frac{1.5}{2}$, $\frac{3}{6.5} = \frac{6}{13} = \frac{1.5}{3.25}$, and $\frac{4}{6.5} = \frac{8}{13} = \frac{2}{3.25}$

The side-length ratios for Triangle C: 3 to 5, 3 to 6, and 5 to 6.
The corresponding side length ratio for Triangle D are 1.5 to 2.5, 1.5 to 3, and 2.5 to 3.

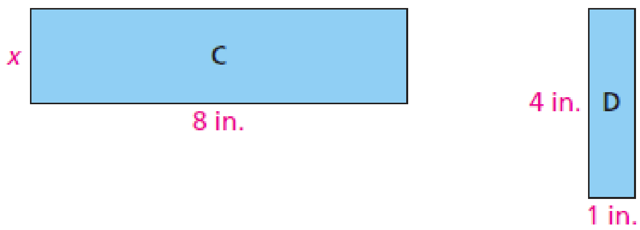
When simplified, the ratios for triangle A and triangle B are equivalent: $\frac{3}{5}=\frac{1.5}{2.5}$, $\frac{3}{6}=\frac{1.5}{3}$, and $\frac{5}{6}=\frac{2.5}{3}$

c. Possible answer: The scale factor from A to B is 1/2. The scale factor from C to D is1/2. The scale factors of these similar triangles tell how many times as great the corresponding side lengths or perimeter are from one figure to a similar figure. The ratios of adjacent side lengths within one triangle tells how many times as great one side length of the triangle is to another side length in the same triangle.

d. In similar triangles, corresponding angles are congruent. This is different from corresponding side lengths because corresponding side lengths vary by a consistent scale factor.

Investigation 4: *Similarity and Ratios*
ACE #11

Rectangles C and D are similar.



a. What is the value of x?
b. What is the scale factor from Rectangle C to Rectangle D?
c. Find the area of each rectangle. How are the areas related?

a. If we compare corresponding sides, the scale factor from Rectangle D to Rectangle C is 2. Therefore, we have to multiply the width of Rectangle D by 2 to find the width of Rectangle C. We get x = 2 inches

b. 0.5

c. The area of C is 16 square inches. The area of D is 4 square inches. The area of D is 1/4 the area of C. The factor 1/4 is found by taking the square of the scale factor, $(½)^2$.