

Comparing and Scaling: Homework Examples from ACE

Investigation 1: *Ways of Comparing: Ratios and Proportions*, ACE #5, #33

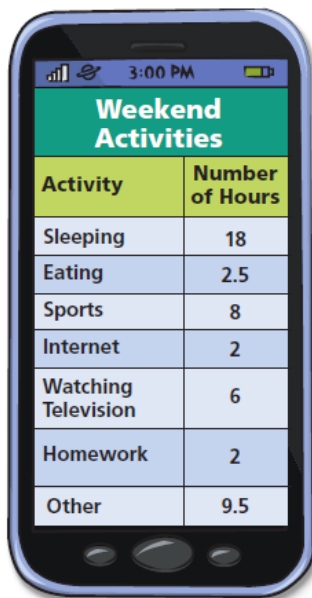
Investigation 2: *Comparing and Scaling Rates*, ACE #8, #9, #15

Investigation 3: *Using Ratios, Percents, and Proportions*, ACE #11 #34

Investigation 1: *Ways of Comparing: Ratios and Proportions*

ACE #5

Students at a middle school are asked to record how they spend their time from midnight on Friday to midnight on Sunday. Carlos records his data on his phone.



Activity	Number of Hours
Sleeping	18
Eating	2.5
Sports	8
Internet	2
Watching Television	6
Homework	2
Other	9.5

Decide whether each statement is an accurate description of how Carlos spent his time that weekend. Explain your reasoning.

- Carlos spent one-sixth of his time watching television.
- The ratio of hours spent watching television to hours spent doing chores or homework was 3 to 1.
- Sports, Internet, and watching television took about 33% of his time.
- Time spent doing homework was only 20% of the time spent watching television.
- Sleeping, eating, and “other” activities took up 12 hours more than all other activities combined.

a. The “whole” in this case is “his time,” so first we have to calculate the total number of hours involved. This is 48 hours. Since $(\text{TV time}) : (\text{total time}) = 6 : 48$ or $1 : 8$ this is not an accurate statement.

b. TV time: chores time = $6 : 2 = 3 : 1$. This is an accurate statement.

c. Recreation + phone time + TV time = 16 hours. His total time = 48 hours. $16 : 48 = 1 : 3$, or 33%. This is an accurate statement.

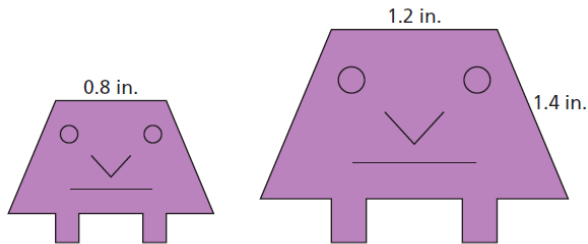
d. Chores time: TV time = $2 : 6 = 1 : 3$ or about 33%. This is not an accurate statement.

e. Sleeping + eating + “other” = 30 hours. This leaves $(48 - 30)$ hours left. 30 hours is 12 hours more than 18 hours. This is an accurate statement.

Note: The way of comparing in part e involves a subtraction to find a difference. Ratios involve dividing to find a factor or rate.

Investigation 1: *Ways of Comparing: Ratios and Proportions*
ACE #33(Connections)

The sketches below show two members of the Grump family. The Grumps are geometrically similar.



Write statements comparing the lengths of corresponding segments in the Grumps. Use each concept at least once.

- a. ratio
- b. fraction
- c. percent
- d. scale factor

a. The top of the little Grump: top of the large Grump = $0.8:1.2$ or, scaling this up, $8:12$ or $2:3$. (This is the only comparison possible, since the question specifies corresponding parts.)

b. The lengths in the smaller drawing are $\frac{2}{3}$ of the lengths in the larger drawing (or the lengths in the larger drawing are $\frac{3}{2}$ or 1.5 times the lengths in the smaller drawing.)

c. The lengths in the smaller drawing are 66.7% of the lengths in the larger drawing (or the lengths in the larger drawing are 150% of the lengths in the smaller drawing).

d. The scale factor from the small to the large drawing is $\frac{3}{2}$ or 1.5 , which means that if you multiply lengths in the small drawing by 1.5 you get the lengths in the larger drawing. (Or we could say that the scale factor from the large to the small drawing is $\frac{2}{3}$, which means that multiplying lengths in the large drawing by $\frac{2}{3}$ will produce the lengths in the small drawing.)

Note: When the order in the English sentence about scale factor mentions the small drawing first, and uses the word “to,” students may start to write the length on the small drawing first, and come up with the wrong scale factor. It may help to use an arrow to indicate the action of the scale factor.

Length on small drawing: length on large drawing = $0.8:12 = 2:3$.

BUT

Small drawing to large drawing, indicates we want to know what factor will change $0.8 \rightarrow 1.2$ or $2 \rightarrow 3$. The arrow represents the action of multiplying by the scale factor 1.5 .

Investigation 2: *Comparing and Scaling Rates*

ACE #8

At camp, Miriam uses a pottery wheel to make 3 bowls in 2 hours. Duane makes 5 bowls in 3 hours.

- Who makes bowls faster, Miriam or Duane?
- How long will it take Miriam to make a set of 12 bowls?
- How long will it take Duane to make a set of 12 bowls?

a. One way to make a comparison is to scale these rates up so that either the number of bowls or the time is the same for both Miriam and Duane.

Miriam makes 3 bowls in 2 hours, or 6 bowls in 4 hours, or 9 bowls in 6 hours, or 12 bowls in 8 hours, or 15 bowls in 10 hours etc.

Duane makes 5 bowls in 3 hours, or 10 bowls in 6 hours, 15 bowls in 9 hours.

This means that Miriam takes longer to make 15 bowls than Duane. Duane is faster.

(If we make the times equal, then Miriam makes 9 bowls in 6 hours, while Duane makes 10 bowls in 6 hours. So Duane is faster.)

b. Miriam makes 12 bowls in 8 hours.

c. Scaling up Duane's ratio 5 bowls: 3 hours by multiplying by whole numbers does not produce 12 bowls: whole number of hours.

We have to think about $5:3 = 12:x$. Students might do this by representing the ratios as fractions.

$\frac{5}{3} = \frac{12}{x}$ and then renaming the fractions so that both numerators are 60.

$\frac{5}{3}$ becomes $\frac{60}{36}$.

$\frac{12}{x}$ becomes $\frac{60}{5x}$.

So $5x = 36$, so Duane takes $\frac{36}{5}$ or $7\frac{1}{5}$ hours to make 12 bowls.

(Students might also solve this by changing 3 hours into 180 minutes and then making a unit rate for Duane of 1 bowl every 180 5 minutes, that is 36 minutes.)

Investigation 2: Comparing and Scaling Rates

ACE #9

The dairy uses 50 pounds of milk to make 5 pounds of cheddar cheese.



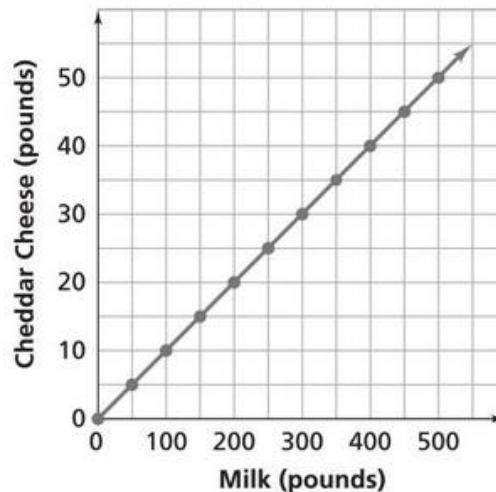
- Make a rate table showing the amount of milk needed to make 5, 10, 15, 20, . . . , and 50 pounds of cheddar cheese.
- Graph the relationship between pounds of milk and pounds of cheddar cheese. First, decide which variable should go on each axis.
- Write an equation relating pounds of milk m to pounds of cheddar cheese c .
- What is the constant of proportionality in your equation from part (c)?
- Explain one advantage of each method (the graph, the table, and the equation) to express the relationship between milk and cheddar cheese production.

a (rate table) and b (graph)

Milk Needed to Make Cheddar Cheese

Cheese (pounds)	Milk (pounds)
5	50
10	100
15	150
20	200
25	250
30	300
35	350
40	400
45	450
50	500

Milk Needed to Make Cheddar Cheese



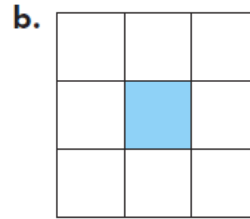
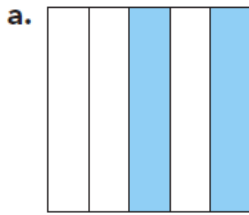
c. $\frac{1}{10}m = c$ or $m = 10c$.

d. $\frac{1}{10}$ for the equation $\frac{1}{10}m = c$ and 10 for the equation $m = 10c$.

e. Possible answers: The graph visually shows the relationship between amounts of milk and cheese. The table allows one to look up how much milk is needed to yield any given amount of cheese. The equation allows for quick calculation of the amount of milk needed for any amount of cheese.

Investigation 2: *Comparing and Scaling Rates*
 ACE #15 (Connections)

For each diagram, write three statements comparing the areas of the shaded and unshaded regions. In one statement, use fractions to express the comparison. In the second, use percentages. In the third, use ratios.



The question asks for a comparison of shaded:unshaded, not shaded: total.

a. The shaded area is $\frac{2}{3}$ the size of the unshaded area.

The shaded area is 66.7% of the unshaded area; the shaded area:unshaded area = 2:3.

b. The shaded area is $\frac{1}{8}$ of the unshaded area.

The shaded area is 12.5% of the area of the unshaded part. Shaded area: unshaded area = 1:8.

Investigation 3: *Using Ratios, Percents, and Proportions*
 ACE #11

Find the missing values in the table.

Costs and Revenue for Roberto's Sales

Buying Price	Markup (80% of buying price)	Selling Price	Commission (25% of markup)	Profit (money the shop makes on the sale)
\$100	\$80	\$180	\$20	\$60
\$10	■	■	■	■
\$55	■	■	■	■
\$125	■	■	■	■

Student strategies for finding the missing values may vary. Students may set up a proportion or they may take the percentages.

Costs and Revenue for Roberto's Sales

Buying Price	Markup (80% of buying price)	Selling Price	Commission (25% of markup)	Profit (money the shop makes on the sale)
\$100	\$80	\$180	\$20	\$60
\$10	\$8	\$18	\$2	\$6
\$55	\$44	\$99	\$11	\$33
\$125	\$100	\$225	\$25	\$75

Investigation 3: *Using Ratios, Percents, and Proportions*

ACE #34

Lealani has 24 scoops of high-fiber chimp food.

a. How many scoops of high-protein food should Lealani add to the mix if she wants to give it to baby chimps? Recall that baby chimps need 40, high-fiber food and 60, high-protein food.

b. How many scoops of high-protein food should Lealani add to the mix if she wants to give it to adult chimps? Recall that adult chimps need 60, high-fiber food and 40, high-protein food.

a. 36; $24 \times 1.5 = 36$

Note: The unit rate of high-protein food to high-fiber food is 1.5.

b. 16; $24 \times \frac{2}{3} = 16$

Note: The unit rate of high-protein food to high-fiber food is $\frac{2}{3}$.

