Looking for Pythagoras: Homework Examples from ACE

Investigation 1: *Coordinate Grids,* ACE #20, #37
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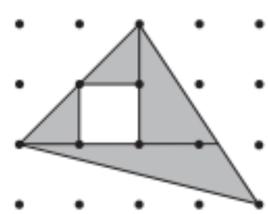
Investigation 1: *Coordinate Grids*
ACE #20

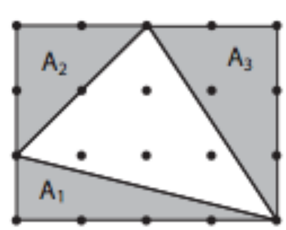
20. Find the area of each triangle. If necessary, copy the triangles onto dot paper.

Students know that the area of a triangle can be found by using the formula Area = ½ (base × height). (See *Covering and Surrounding*.) For this problem, the challenge is that the base length or the height are not immediately clear. When students know the Pythagorean Theorem, they will use it to find these lengths, but for now they need a different strategy.

20. Students commonly use 2 different strategies to find areas: they subdivide the area into shapes for which they know the area; or they surround the shape by a rectangle and subtract areas from the rectangle.
Subdividing the area as shown below may not be very helpful if the areas of the shaded triangular shapers are not easy to find.



Surrounding the triangle with a 4-by-3 rectangle (see below) and then subtracting the shaded areas (using the formulas for the area of a triangle) gives the following calculation:



(Area of the un-shaded triangle) = (area of 4-by-3 rectangle) – ($A_1 + A_2 + A_3$)
= 12 – (2 + 2 + 3)
= 5 square units.

Investigation 1: *Coordinate Grids*
ACE #37

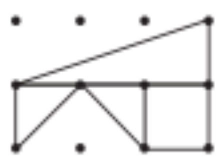
37. Marcia finds the area of a figure on dot paper by dividing it into smaller shapes. She finds the area of each smaller shape and writes the sum of the areas as ½ (3) + ½ + ½ +1.

  a. What is the total area of the figure?
  b. On dot paper, draw a figure Marcia might have been looking at.

37.

  a. ½ (3) + ½ + ½ +1= 3.5 square units
  b. This problem makes students attend to the format of the expression. This develops student symbol sense.
     There seem to be 4 areas summed together in the expression ½ (3) + ½ + ½ +1 in which each term of the expression represents a geometric figure. The term "½ (3)" implies a triangle with the area ½ (base × height) where the base could be 3 and the height could be 1 (see below). The terms "½ " each imply a triangle with the area ½ (base × height) where the base could be 1 and the height could be 1 (see below). The term "1" could be a square with side 1.

     Putting all these clues together, one of many possible figures could be:



Investigation 2: *Squaring Off*
ACE #16

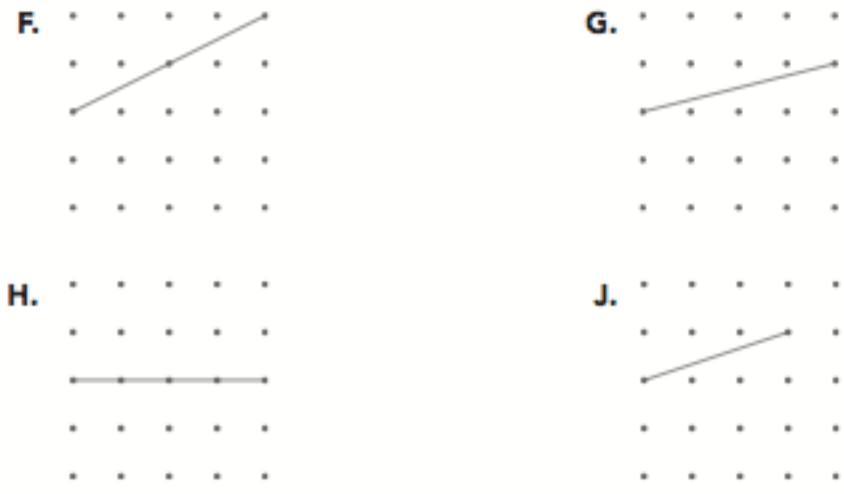
Tell whether each statement is true.
16. $11 = \sqrt{(101)}$

Students learn in this Investigation that to find the area of a square they must multiply the length of a side by itself, AND, to find the length of the side of a square from its area, they must find the *square root* of the area.

16. This question asks: if a square has area 101 square units then is its side 11 units long?
  $11^2 = 121$, so if a square had side length 11 units then its area is 121 square units.
  Therefore, $11 = \sqrt{(101)}$ is not true.
  Since $10^2 = 100$, $\sqrt{(100)} = 10$; and since $11^2 = 121$, $\sqrt{(121)} = 11$, then $\sqrt{(101)}$ must lie between 10 and 11.

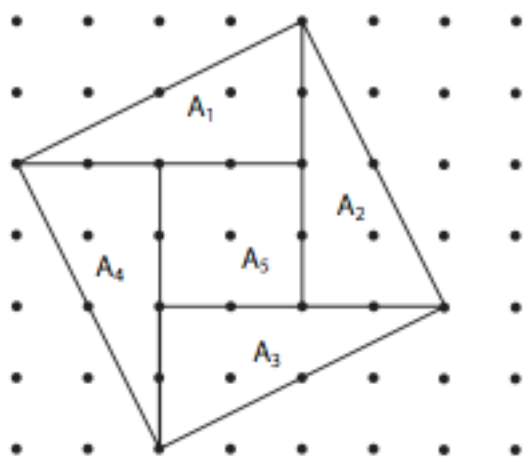
Investigation 2: *Squaring Off*
ACE #44

44. **Multiple Choice.**
Which line segment has a length of √17 units?



44. We need to check each figure to find which segment is the side of a square with area 17 square units.
Figure F shows a slanting line segment, which can be thought of as the side of a square. If we build the rest of the square we see it has the area 20 square units.

$A_1 + A_2 + A_3 + A_4 + A_5 = 4 + 4 + 4 + 4 + 4 = 20$ square units.
Therefore, the line segment is 20 units long.

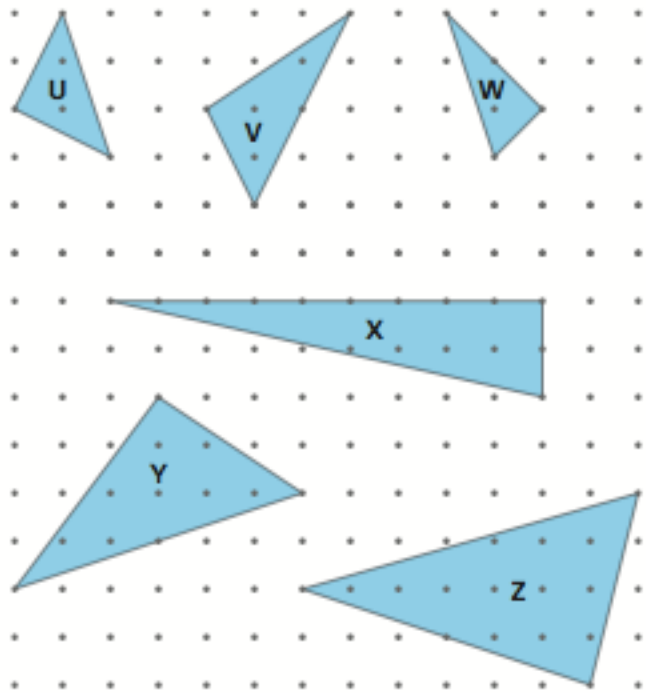


Investigation 2: *Squaring Off*
ACE #65

65.

a. Which of the triangles below are right triangles?

b. Find the area of each right triangle.



At this stage students cannot use the Pythagorean theorem to show whether these triangles are right angled or not. But they can use the strategy they have been using to draw a square to decide if two sides of the triangle are perpendicular. In Investigation 1 of this unit, in order to create a square on a slanting line segment, students discussed the use of slope (See *Moving Straight Ahead).* If the given line segment has slope $^{a}/_{b}$, then the slope of the adjacent side of the square has slope $^{-b}/_{a}$. (Students may not have formalized this yet.)

65.

a. For figure U we can see that the slopes of the bolded line segments are $^{2}/_{1}$ and $^{-1}/_{2}$. Therefore, these two line segments are perpendicular. This is a right triangle.

b. Area of figure U can be found by subdividing or surrounding (see ACE 20 Investigation 1). Or it can be found by noticing that Figure U is half of a square with area 5 square units (not drawn here). So area of figure U = 2.5 square units.

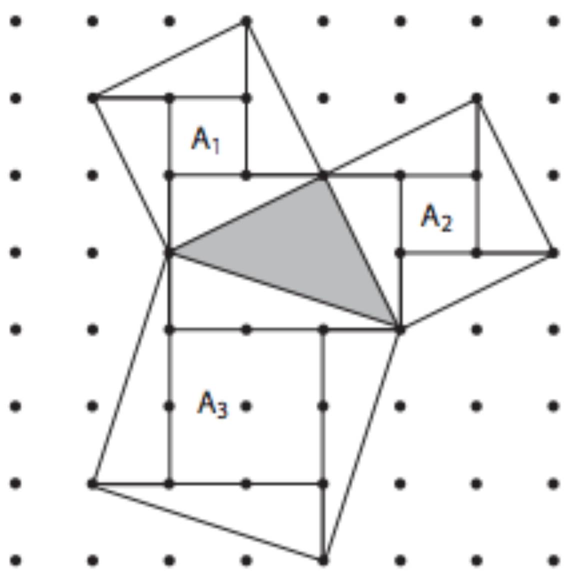
The other triangles can be investigated the same way.

Investigation 3: *The Pythagorean Theorem*
ACE #2

2. The triangle below is a right triangle. Show that this triangle satisfies the Pythagorean Theorem.

Students might approach this problem either by showing that the triangle is right angled, in which case the Pythagorean Theorem applies. Or, they might find the areas of the squares on the sides, and check that these fit the Pythagorean relationship. If they take the first approach they must have a way to show that two sides of the triangle are perpendicular. The relationship between the slopes of perpendicular lines has not been formally stated in any unit thus far (will be formalized in Shapes of Algebra), but some classes may have drawn a conclusion about this relationship in this unit.

The second approach is illustrated below.



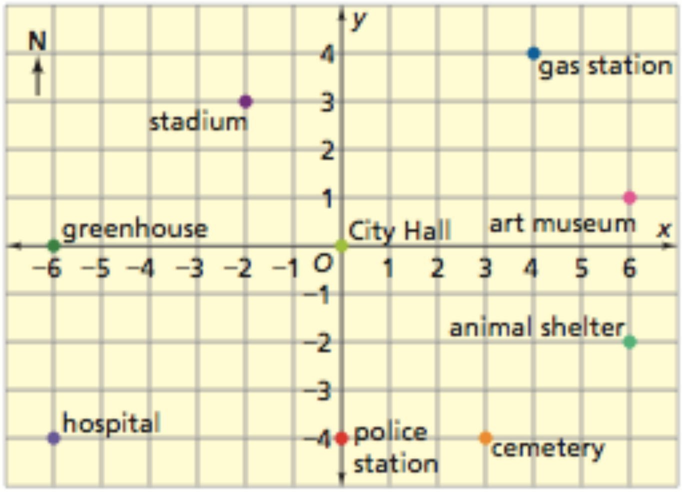
The area A_3 can be found by subdividing the square.

$A_3 = 1.5 + 1.5 + 1.5 + 1.5 + 4 = 10$ square units.

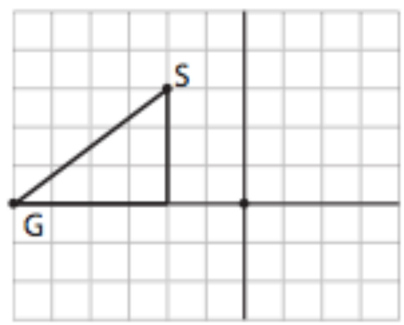
Likewise, $A_2 = 5$, and $A_1 = 5$ square units. So, $A_1 + A_2 = A_3$.

Investigation 3: *The Pythagorean Theorem*
ACE #9

9. Find the flying distance in blocks between the two landmarks, Greenhouse and Stadium, without using a ruler.



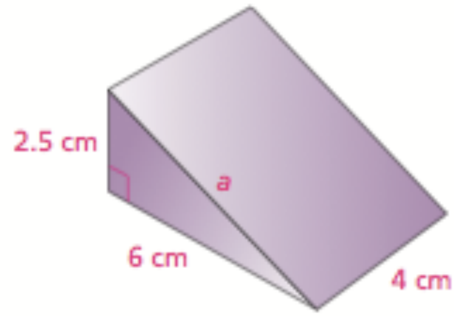
The segment joining the two landmarks can be thought of as the hypotenuse of a right triangle, with legs of lengths 4 and 3 units.



$GS^2 = 3^2 + 4^2 = 25$. Therefore, GS = 5 units. The distance between the greenhouse and the stadium is 5 blocks.

Investigation 3: *The Pythagorean Theorem*
ACE #17

17. The prism has a base that is a right triangle



a. What is the length ***a***?

b. Do you need to know the length ***a*** to find the volume of the prism? Do you need to know it to find the surface area? Explain.

c. What is the volume?

d. What is the surface area?

17.

a. Applying the Pythagorean Theorem $2.5^2 + 6^2 = a^2$. So, $42.25 = a^2$. So, a = 6.5 cm.

b. To find the volume you need to know the area of the base of the prism and the height of the prism. The area of the base is 0.5 (2.5 x 6) = 7.5 square units. You don't need to know *a* to find this base area. To find the surface area you need to know the areas of all faces. The triangular base areas can be found as above. But the area of one of the rectangular faces is *4 x a* square units. So we do need to know *a* to find surface area.

c. 30 cm^3; $0.5\ (6 \times 2.5) \times 4 = 30$

d. 75 cm^2; $(2.5 \times 4) + 2\ [\ 0.5\ (6 \times 2.5)\ ] + (6 \times 4) + (6.5 \times 4) = 10 + 15 + 24 + 26 = 75$

e. The net should show 3 rectangular faces and 2 triangular faces.

Investigation 4: *Using the Pythagorean Theorem: Understanding Real Numbers*
ACE #6

6. Write each fraction as a decimal. Tell whether the decimal is *terminating* or *repeating*. If the decimal is repeating, tell which digits repeat.

$$\frac{4}{99}$$

6. In an earlier unit, *Let's Be Rational*, students learned to think of a fraction in different ways; for example, a fraction might be thought of as parts out of a whole, or as a ratio, or as a division. The last interpretation helps to connect decimals to fractions.

$$\begin{array}{r} .040404 \\ 99\overline{)4.000000} \\ \underline{396} \\ 400 \\ \underline{396} \\ 400 \\ \underline{396} \end{array}$$

$^4/_{99}$ = 0.040404...
The decimal is repeating, and the digits that repeat are "04".

Note: Every fraction can be written as a decimal by dividing as above. Since there can be a finite number of choices for the remainders created by such a division we eventually come to a situation where the remainder is zero, in which case the decimal terminates, or a previous remainder repeats, in which case the decimal answer repeats. For example, when dividing by 99 we could theoretically have any remainder from 0 to 98. After all remainders have been used once one of them must repeat. In fact the only remainder created by the above division is 4, and so the division process repeats very quickly.

Investigation 4: *Using the Pythagorean Theorem: Understanding Real Numbers*
ACE #34

Estimate the square root to one decimal place *without* using the $\sqrt{}$ key on your calculator. Then, tell whether the number is *rational* or *irrational*

34. $\sqrt{15}$

34. We can use the perfect squares that we know to find an estimate for $\sqrt{15}$. We know the perfect squares of 9 and 16, and that $9 < 15 < 16$. So, $\sqrt{9} < \sqrt{15} < \sqrt{16}$.

$\sqrt{16} = 4$.

$\sqrt{9} = 3$.

So, $3 < \sqrt{15} < 4$.

We can see that $\sqrt{15}$ is closer to $\sqrt{16}$ than to $\sqrt{9}$. Therefore, we might try 3.9 as a first approximation.

$3.9^2 = 15.21$.

$3.8^2 = 14.44$.

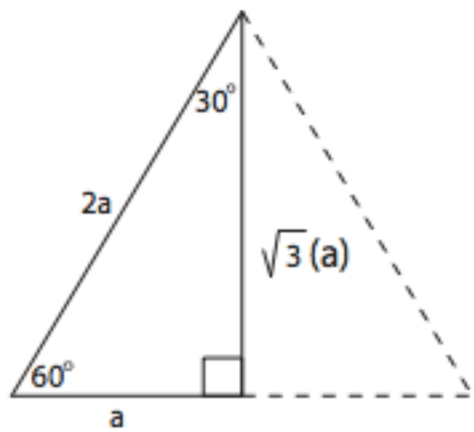
So, 3.9 appears to be a better approximation than 3.8.

Since there is not exact decimal answer for $\sqrt{15}$ it is an irrational number (that is, the decimal answer neither terminates nor repeats).

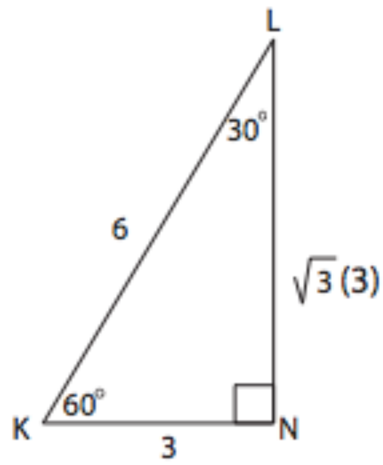
Investigation 5: *Using the Pythagorean Theorem: Analyzing Triangles and Circles*
ACE #7

7. Find the perimeter of triangle KLM.

7. Solution.
In this Investigation students applied the Pythagorean theorem to a particular triangle, with angles 30, 60 and 90 degrees. By observing that this triangle is half of an equilateral triangle they were able to conclude that the shortest side is always half of the hypotenuse; and by applying the Pythagorean Theorem they were able to conclude that the longer side is always √3 times the shortest side. These relationships apply to any 30-60-90 triangle, because all such triangles are similar, or scale copies of each other. A general 30-60-90 triangle is pictured below:



Applying this to triangle KLN we have:



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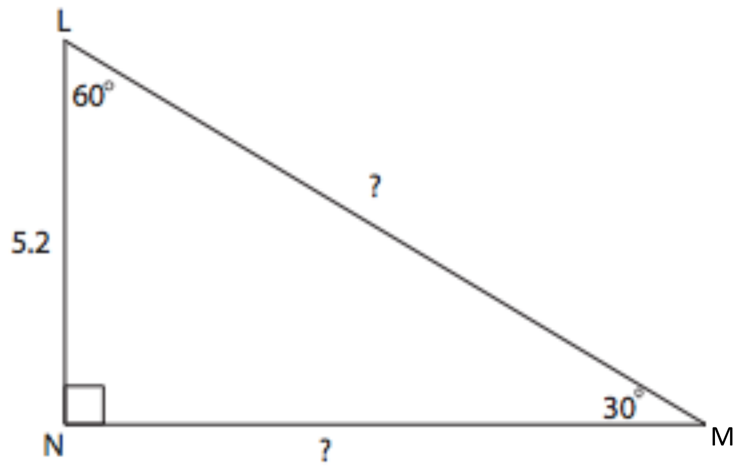
Now look at triangle MLN. It is also a 30-60-90 triangle, and we know the shortest side is √3(3) units, or approximately 5.2



We can deduce the length of hypotenuse LM and longer leg MN by using the length of the shortest side LN.
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