Mathematics Background

Building Proficiency Using Algorithms

Rational numbers are the heart of the middle-grade experiences with number concepts. Understanding and using fractions and operations on fractions can be challenging for students as they transition from elementary into the middle grades. Students will have had some experiences with addition, subtraction, and multiplication with rational numbers in earlier grades. They are unlikely, however, to be proficient with these operations, in particular in choosing and using the appropriate operation or sequence of operations to solve a problem.

Operations

Addition	$3 + 1 = 4$	Subtraction	$5 - 3 = 2$
Multiplication	$3 \times 2 = 6$	Division	$8 \div 4 = 2$

Students may become confused about rational numbers if they rush into symbol manipulation with fraction operations. They need to spend time making sense of the concepts and building experiences that show reasons for why the algorithms work. In addition, students need to understand the operations in ways that help them judge what operation or combination of operations is needed in a given situation.

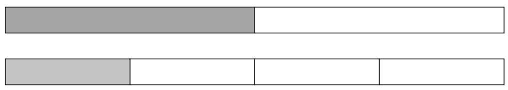
Let's Be Rational is designed to provide experience in building algorithms for the four basic operations with fractions, as well as opportunities for students to consider when such operations are useful in solving problems. Building this kind of thinking and reasoning supports the development of skill and fluency with the algorithms. By the end of this Unit, expect students to understand and be able to use efficient algorithms for all four basic operations with fractions, including mixed numbers.

For all four operations we use the same type of development.

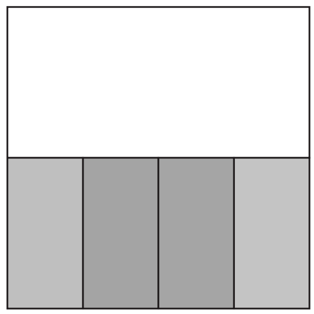
- The development of algorithms for each operation and the understanding of those algorithms involve experiences with contextual problems, models, strategies, and estimation. Problems in context help students make sense of an operation and how the operation can be computed.
- The contexts lead students to model situations and write number sentences that are representative of a particular situation, so they begin to understand when an operation is appropriate. By analyzing the diagrams and models they develop and their resulting number sentences, and relating this to their symbolic work, students develop algorithms for fraction operations.
- Important goals of all this work are for students to write and read mathematical language and to make decisions using what they know.

This development of algorithms draws upon concepts and procedures developed in elementary grades and *Comparing Bits and Pieces*. In *Comparing Bits and Pieces,* students developed an understanding of basic interpretations, models, equivalence, and ordering of rational numbers. In *Let's Be Rational,* they draw upon this development and the models that were introduced—fraction strips or bars, number lines, grid area, and partition models—because these concepts connect directly to operations on rational numbers. (See Mathematics Background for *Comparing Bits and Pieces* for a full discussion of the concepts and models introduced here.)

Fraction strips



Partition model



Choosing Appropriate Operations

Estimation

The initial question CMP helps students ask is, "About how big will the answer be? What answer makes sense?" Having a ballpark estimate gives students a way to know if their computations are sensible, whether the calculation has been done by hand or with a calculator. At this point in the curriculum, students have had quite a bit of practice finding equivalent fractions and decimals and changing forms of fractions. They have developed some benchmark fractions that they can use to estimate relative size and to check their computations.

The strategies used to estimate can differ. In a situation in which the goal is to decide what whole number a sum of fractions is closest to, rounding each fraction to the nearest whole number or benchmark fraction, for instance a half, is a useful strategy. In some contexts, you may want to use an estimation strategy that leads to an estimate that you know is too large (overestimate) or too small (underestimate). As a general rule, to be sure you have enough, you overestimate what you need, and you underestimate what you have or what you can do. This can be applied to shopping and other contexts, such as ingredients for cooking and materials for home projects.

Overestimate what you need.	Underestimate what you can do.
You need to make a frame. $10\frac{3}{8}$ in. by $17\frac{7}{8}$ in. Use benchmarks to estimate. $10\frac{1}{2} + 10\frac{1}{2} + 18 + 18 = 57$ You need 57 inches of wood, and you can round up to 60 inches.	You hiked $12\frac{7}{8}$ mi in $4\frac{3}{4}$ h. Use benchmarks to estimate. $12\frac{1}{2} \div 5 = 2\frac{1}{2}$ You can hike at a speed of $2\frac{1}{2}$ miles per hour.

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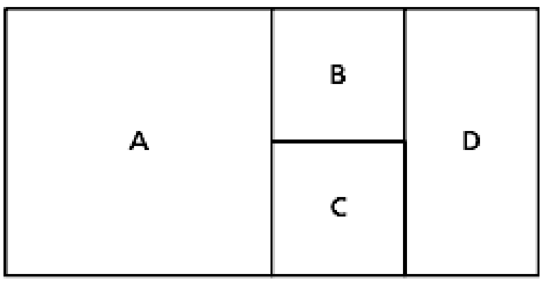
Building and Extending Addition and Subtraction Fluency

We build on and extend students' earlier experiences with addition and subtraction of fractions to connect with contexts, models, and diagrams. Addition and subtraction of fractions serve as a foundation for exploration of multiplication and division, both of which use similar models and diagrams. Strategies for operations with fractions can be developed with contexts that help students learn how to put fractions together and take them apart.

As students model and symbolize aspects of contextual situations, they develop meaning for and skill in using the operations of addition and subtraction. As well, students learn the value of equivalence when changing the representation of fractions to a form with a common denominator.

> ### Example
>
> **Common Denominators**
>
> | Addition | Subtraction |
> |---|---|
> | $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ | $\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$ |

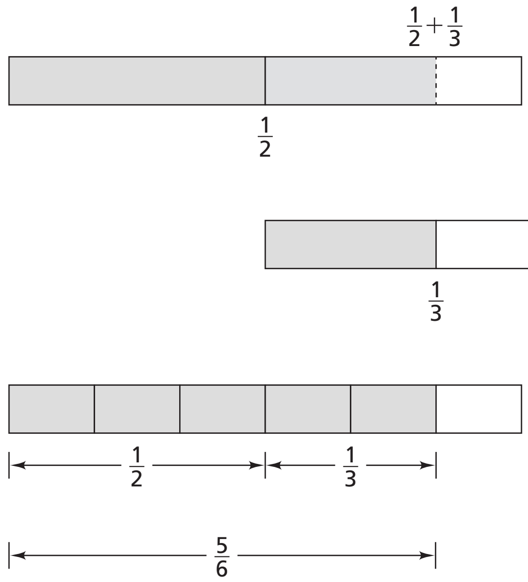
Students' previous work with equivalence and partitioning is critical to the development of strategies for adding and subtracting. The following area model provides a context in which both naming fractional parts of a whole and equivalence can emerge as students try to write number sentences to model combining square *A* with square *B*. Visit Teacher Place at mathdashboard.com/cmp3 to see the image gallery.



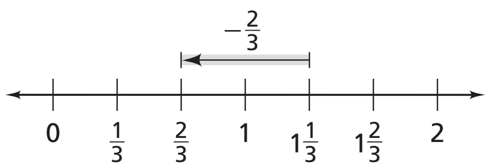
Find the sum of A and B.

In order to find the sum of $A + B$, or $\frac{1}{2}+\frac{1}{8}$ of the whole, students need to use equivalent fractions to rename $\frac{1}{2}$ as $\frac{4}{8}$. The area model helps students visualize *A*, which is $\frac{1}{2}$, as 4 eighth-size sections. By asking students to write number sentences, and to explain how the number sentence helped them arrive at the sum $\frac{5}{8}$, students begin to understand why it is necessary to rename fractions when adding and subtracting and the role that equivalence plays in doing so.

There are other models that can be used to highlight the role of equivalence and support understanding of addition and subtraction. The fraction-strip model was used in conjunction with the number-line model in *Comparing Bits and Pieces* to develop meaning for fractions and equivalence. Here fraction strips are used to represent $\frac{1}{2} + \frac{1}{3} = \frac{5}{8}$.



The number-line model helps make the connection to fractions as numbers or quantities. This is a number line for 0 to 2 marked to illustrate $1\frac{1}{3} - \frac{2}{3} = \frac{2}{3}$.

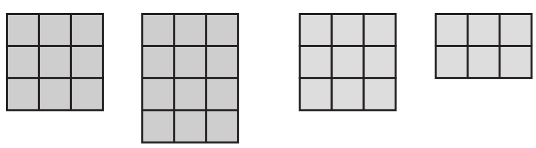


Multiplication of Rational Numbers

One of the first hurdles for students in understanding multiplication of fractions is realizing that multiplication does not always "make larger," as their experience with whole number multiplication has firmly established. In fact, with multiplication of a fraction by a whole number, the fraction can be interpreted as an operator that may stretch (make larger) or shrink (make smaller) depending on whether the fraction is greater or less than 1. This is an important aspect of multiplication of fractions.

Stretching $9 \times \frac{4}{3} = 12$

Shrinking $9 \times \frac{2}{3} = 6$



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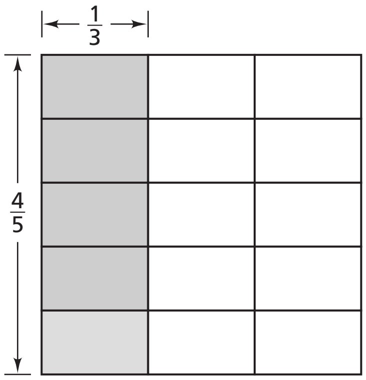
A second hurdle for students is understanding that when they encounter a situation where one needs to take a fraction of a quantity, *of* means multiplication. For example, to find $\frac{2}{3}$ of 9, you multiply $\frac{2}{3} \times 9$ to get 6. Avoid the temptation to start by telling students this rather than have them see it for themselves. In discovering what *of* means in this context, student learning is enhanced.

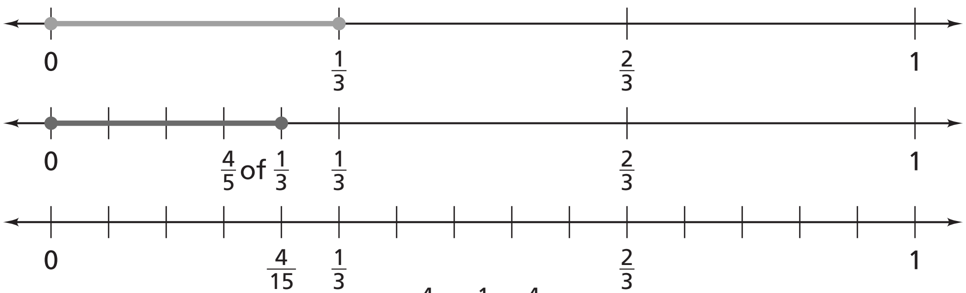
Words	Mathematics
two thirds *of* nine	$\frac{2}{3} \times 9$

Models for multiplication of fractions used in *Let's Be Rational* are both area models and partitioning. (Area models are also useful for representing multiplication of decimals and percents.) The area model below shows $\frac{4}{5}$ of $\frac{1}{3}$.



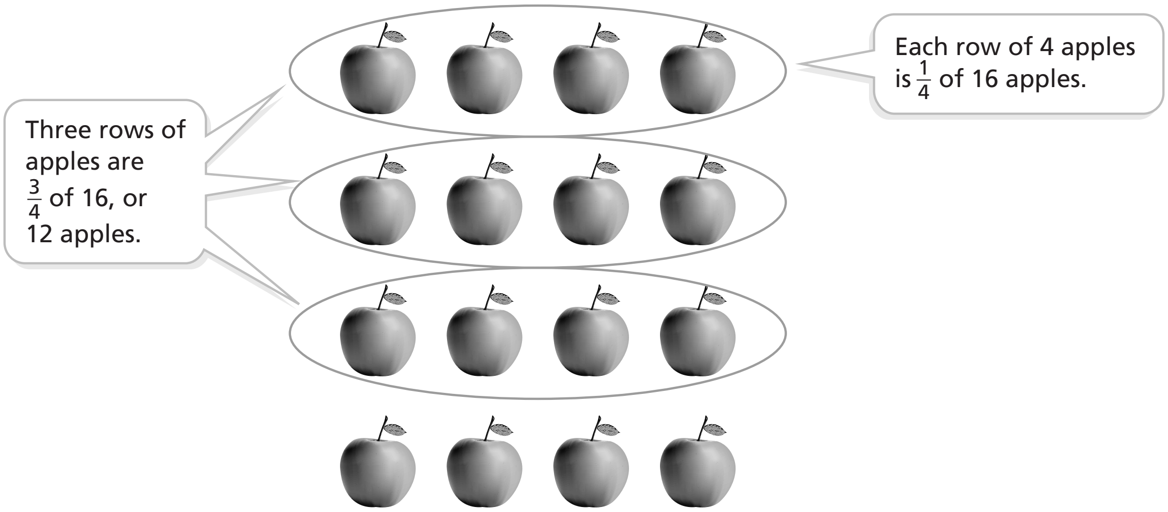
$\frac{4}{5}$ of $\frac{1}{3} = \frac{4}{15}$

Students also use a model of fraction situations that is based on partitioning a number line or strip. The number lines below show finding $\frac{4}{5}$ of $\frac{1}{3}$, or $\frac{4}{5} \times \frac{1}{3}$.

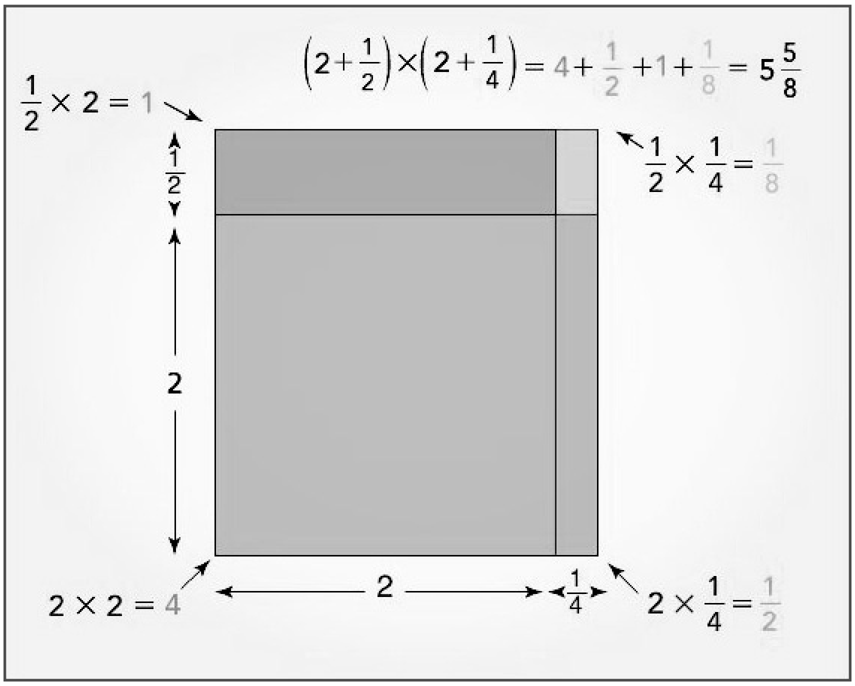


$\frac{4}{5}$ of $\frac{1}{3} = \frac{4}{15}$

You may see students use discrete models to make sense of situations where the objects they are working on are separate objects. An example of a discrete situation is finding $\frac{3}{4}$ of 16 apples. Here each apple represents a separate entity.



Discrete models can also represent mixed numbers. The animation below shows a discrete model for evaluating $2\frac{1}{2} \times 3\frac{1}{2}$. Visit Teacher Place at mathdashboard.com/cmp3 to see the complete animation.

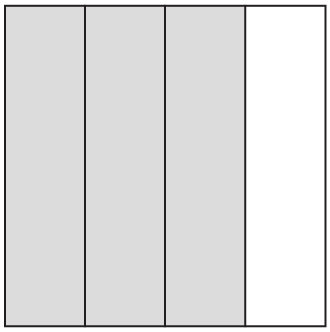


Developing a Multiplication Algorithm

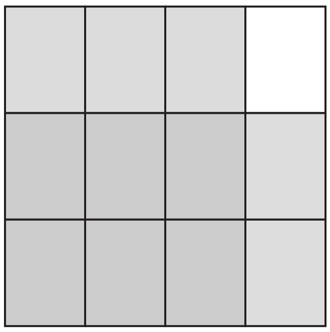
Students notice that multiplication is easy for proper fractions because they can just multiply the numerators and multiply the denominators. However, they have little understanding of why this works. The following paragraphs examine both the area and the number-line model as aids to understanding why the algorithm works.

Area Model

Consider the problem $\frac{2}{3} \times \frac{3}{4}$. To show $\frac{2}{3} \times \frac{3}{4}$ with an area model, first represent the $\frac{3}{4}$ by dividing a square into fourths and shading three of the fourths.

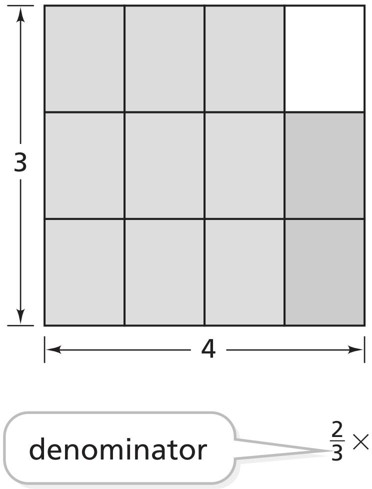


To represent taking $\frac{2}{3}$ of $\frac{3}{4}$, divide the whole into thirds by cutting the square into three rows and then shade two of the three rows. The part where the shaded sections overlap represents the product, $\frac{6}{12}$.



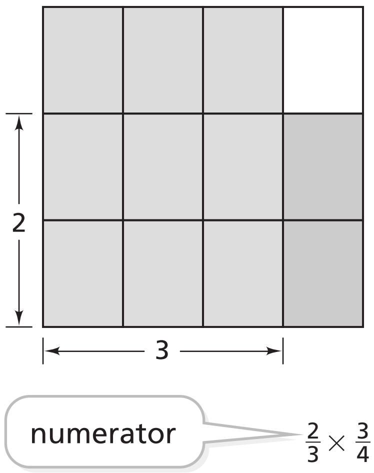
Note what happens to the numerator and the denominator when you partition and how this is related to the algorithm for multiplying fractions. When the square is partitioned, the denominators are used to partition and repartition the whole. In this problem, there are fourths, or four parts. When the fourths are partitioned into thirds, or three parts each, the number of pieces in the whole triples, so there are 12 pieces.

In the algorithm, when you multiply the denominators (3×4), you are resizing the whole to have the correct number of parts. This means that the denominator in the product has the same role as the denominator in a single fraction. The role is to determine how many parts are in the whole.



$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

Likewise, the numerator is keeping track of how many of the one-twelfth parts are in the product. During the original partitioning, $\frac{3}{4}$, or 3 fourth-size parts, were shaded. In order to take $\frac{2}{3}$ of the 3 one-fourth size parts, you have to take 2 of the one-twelfth sections from each of the 3 one-fourth size parts. This can be represented by the product of the numerators, 2×3.



$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

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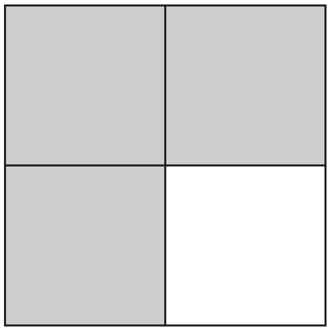
Note that dividing a square with both horizontal and vertical lines to represent the first fraction does not lead to the kind of partitioning that suggests multiplication of numerators and denominators. For example, represent $\frac{3}{4}$ this way.



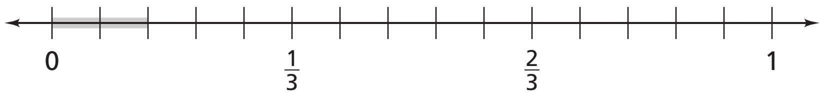
Then you may find $\frac{2}{3}$ of $\frac{3}{4}$ by noticing that there are three pieces shaded and you are concerned with 2 of them, so the answer is $\frac{2}{4}$. This strategy is perfectly reasonable for this problem. The question is whether this strategy will always work for all fractions. For $\frac{1}{5} \times \frac{2}{3}$, this strategy does not help us solve the problem.

Number-line Model

Next, take a look at how the number-line model is helpful for $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$. You can also generalize the model, even if the process is tedious with large numerators or denominators.

Draw a number line and label 0 and 1. Partition the number line into thirds and mark $\frac{1}{3}$ and $\frac{2}{3}$.

Now break each third into 5 equal parts to get a total of 15 equal parts.



Each fifth of a third is $\frac{1}{15}$, so the two parts marked are $\frac{2}{15}$. Again, the product of the numerators gives the numerator of the product and the product of the denominators gives the denominator of the product.

Using Distribution as a Strategy to Multiply Fractions and Mixed Numbers

The Distributive Property is a requirement of the Common Core State Standards for Mathematics. The Distributive Property can be useful when multiplying mixed numbers. However, students often use this strategy incorrectly. Therefore, Problem 3.4 provides an opportunity to use the Distributive Property. It is not the only approach to multiplying fractions, but it is one that is sensible in some situations.

The Distributive Property was introduced in the mathematical context of whole-number multiplication in *Prime Time* and then related to the process of the multiplication of mixed numbers. This approach can help students develop a better understanding of the property and its uses. Here is an algorithm for whole-number multiplication that uses this approach. Consider the expression 32 × 24. You can quickly evaluate it.

$$\begin{array}{r} 32 \\ \underline{\times 24} \\ 8 \\ 120 \\ 40 \\ \underline{600} \\ 768 \end{array}$$

The algorithm you are following is much like multiplying binomials in algebra. It involves breaking up both numbers into their tens and ones place values, as shown explicitly below.

Use distribution to find	32×24
Rewrite the expression.	$(30 + 2) \times (20 + 4)$
Multiply 30 times 20.	$30 \times 20 = 600$
Next, multiply 30 times 4.	$30 \times 4 = 120$
Then multiply 2 times 20.	$2 \times 20 = 40$
Multiply 2 times 4.	$2 \times 4 = 8$
Add the partial products.	$= 768$

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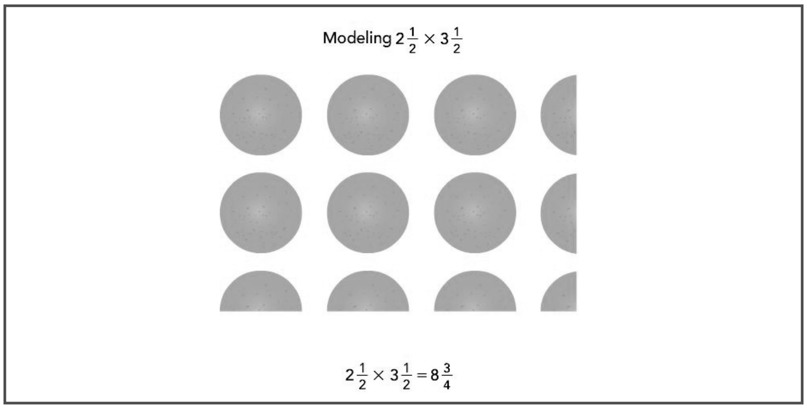
With a problem such as $2\frac{1}{2} \times 2\frac{1}{4}$, students may break up each factor and try to work with $\left(2 + \frac{1}{2}\right) \times \left(2 + \frac{1}{4}\right)$. If they distribute correctly, they can reason through the solution as shown below.

Use distribution to find	$2\frac{1}{2} \times 2\frac{1}{4}$		
Rewrite the expression.	$(2 + \frac{1}{2}) \times (2 + \frac{1}{4})$		
Multiply 2 times 2.	2×2	$= 4$	
Next, multiply 2 times $\frac{1}{4}$.	$2 \times \frac{1}{4}$	$= \frac{1}{2}$	
Then multiply $\frac{1}{2}$ times 2.	$\frac{1}{2} \times 2$	$= 1$	
Multiply $\frac{1}{2}$ times $\frac{1}{4}$.	$\frac{1}{2} \times \frac{1}{4}$	$= \frac{1}{8}$	
Add the partial products.		$= 5\frac{5}{8}$	

The animation here demonstrates an area model approach for multiplying mixed numbers. Visit Teacher Place at mathdashboard.com/cmp3 to see the complete animation.



Another approach that makes sense with this problem is to work with $\left(2 + \frac{1}{2}\right) \times 2\frac{1}{4}$. If you only break up the first factor, the reasoning is as follows.

Use distribution to find	$2\frac{1}{2} \times 2\frac{1}{4}$	
Rewrite the expression.	$(2 + \frac{1}{2}) \times 2\frac{1}{4}$	
Multiply 2 times $2\frac{1}{4}$.	$2 \times 2\frac{1}{4}$	$= 4\frac{1}{2}$
Next, multiply $\frac{1}{2}$ times $2\frac{1}{4}$.	$\frac{1}{2} \times 2\frac{1}{4}$	$= 1\frac{1}{8}$
Add the partial products.		$= 5\frac{5}{8}$

Division of Rational Numbers

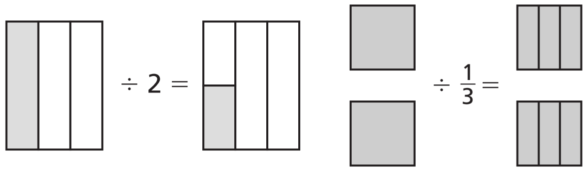
Division presents a number of conceptual difficulties. A quotient involving fractions is not necessarily smaller than the dividend. Again, the size depends on the size of the fraction for both the dividend and the divisor. For example, $3 \div \frac{1}{3} = 9$ and $\frac{1}{4} \div \frac{1}{3} = \frac{3}{4}$ result in a quotient that is larger than the dividend or the divisor. In $\frac{1}{3} \div 9 = \frac{1}{27}$, however, the quotient is smaller than either the dividend or the divisor, and in $\frac{1}{4} \div \frac{3}{4} = \frac{1}{3}$, the quotient is greater than the dividend but less than the divisor.

quotient smaller than dividend

$\frac{1}{3} \div 2 = \frac{1}{6}$

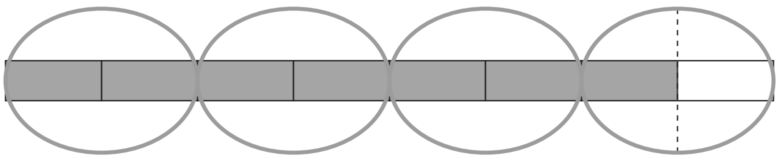
quotient larger than dividend

$2 \div \frac{1}{3} = 6$



Examination of division of fractions in context can help students build an understanding of the operation as well as skill in predicting (or estimating) the kind of answer expected.

In order for students to make sense of any division algorithm, they need to think about what the problem is asking. Creating diagrams to model division problems is a key part of developing this understanding. There are cases where the use of pictorial reasoning is more efficient or just as efficient as an algorithm. Also, the development of an efficient algorithm is tied to the ability to understand pictorially and linguistically what the problem is asking.



As students work toward developing and using algorithms, they may continue to draw pictures to help them think through the problem. However, they also need to learn to talk about what the problem is asking, what the answer means, what makes sense as a solution strategy, and how this language is related to the algorithm.

Our goal is to help students develop an efficient algorithm. Not all students may get to the “reciprocal” algorithm for division of fractions, but they should have efficient strategies that make sense to them to solve problems that call for division with fractions.

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Understanding Division as an Operation

There are two situations associated with division, sharing and grouping situations. To provide support for solving problems, both teachers and students need to understand these two division situations.

Division as a Sharing Situation (partitive model for division)

You can focus on division as a sharing operation in problems such as this one.

> Ms. Li gave peanuts as a prize for a relay race. Suppose the members of the winning team share the peanuts equally among themselves. If four students share $\frac{1}{2}$ pound of peanuts, what fraction of a pound of peanuts does each student get?
>
> **3.3** Sharing a Prize: Dividing a Fraction by a Whole Number

The question is how much each of the four team members will get if the amount is shared equally. You can also think of this as a partitioning, sometimes called partitive, model.

Division as a Grouping Situation (measurement model for division)

Another kind of situation calling for division is a grouping situation. The example below is from Investigation 3.

> Naylah plans to make small cheese pizzas to sell at the school carnival. She has nine blocks of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?
>
> **3.2** Into Pieces: Whole Numbers or Mixed Numbers Divided by Fractions

Here the question is how many groups of size $\frac{1}{3}$ can be made from nine blocks of cheese? Another way to ask this is "How many $\frac{1}{3}$'s are in 9?" This kind of problem has multiple names—measurement, subtractive, or quotitive model. Knowing all of these names is not important for your students, but it is important for teachers to understand them in order to provide experiences with situations representing these different models of division for students. Otherwise students will not have all the tools for deciding when division is the appropriate operation.

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As they master the meaning of operations, we ask students to write problems that fit a given computation expression. This will tell you whether students can interpret different kinds of division situations and whether they can make sense of what the answer to a division problem, including its fractional part, means in a given situation.

Developing a Division Algorithm

Let's Be Rational develops understanding of division of fractions by looking at three cases—division of a whole number by a fraction, division of a fraction by a whole number, and division of a fraction by a fraction. From these situations, several approaches to division are developed: multiplying by the denominator and dividing by the numerator, multiplying by the reciprocal, and the common-denominator approach.

Multiplying by the Denominator and Dividing by the Numerator

When you see a whole number divided by a fraction, such as $9 \div \frac{1}{3}$, it is easiest to interpret this as finding how many $\frac{1}{3}$'s are in 9. To evaluate the expression, students find how many $\frac{1}{3}$'s are in a whole and multiply by 9 to find the total number of $\frac{1}{3}$'s in 9. The reasoning is as follows.

> In $9 \div \frac{1}{3}$, I have to find the total number of $\frac{1}{3}$'s in 9.
> I know that there are three $\frac{1}{3}$'s in 1, so there are 9×3 or 27 of the $\frac{1}{3}$'s in 9.
> In summary, $9 \div \frac{1}{3} = 9 \times 3 = 27$.

The next problem is $9 \div \frac{2}{3}$. A key to understanding division of fractions is the relationship between the two problems $9 \div \frac{1}{3}$ and $9 \div \frac{2}{3}$. The question is how are the answers related? The answer to the first problem is 27 and the answer to the second is $13\frac{1}{2}$. Why does it make sense for the answer to the second to be half that of the first? One student provided the following reasoning.

> You can interpret $9 \div \frac{1}{3}$ as how many $\frac{1}{3}$'s are in 9 and $9 \div \frac{2}{3}$ as how many $\frac{2}{3}$'s are in 9. Now it makes sense that it will take twice as much to make $\frac{2}{3}$ as to make $\frac{1}{3}$. So the number you can make will be half as large.

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The relationship between the two division problems allows students to relate a whole string of division problems, such as $9 \div \frac{1}{3}$, $9 \div \frac{2}{3}$, $9 \div \frac{3}{3}$, and $9 \div \frac{4}{3}$. Here we think of division of fractions as multiplying by the denominator of the divisor to find how many are in one whole and then dividing by the numerator because that is how many it takes to make a piece of the required size. These two actions are the same as multiplying by the reciprocal.

Key Concept

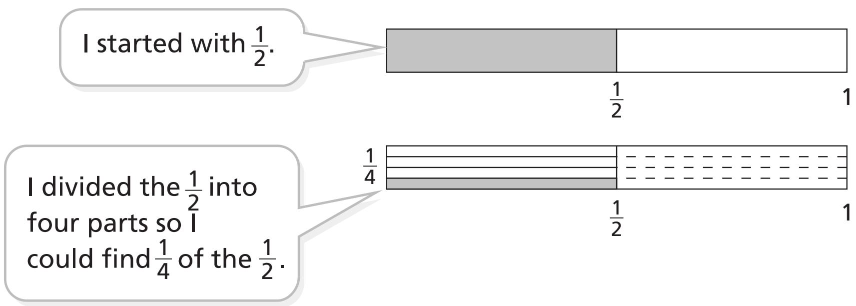
Multiplying by the denominator and dividing by the numerator of a fraction is the same as multiplying by the reciprocal of a fraction.

Many students are able to see the pattern of multiplying by the denominator and dividing by the numerator of the divisor and explain why it makes sense through the following kind of classroom talk.

> For $\frac{2}{3} \div \frac{3}{4}$, you can find the quotient by multiplying $\frac{2}{3}$ by $\frac{4}{3}$. The way I think about it, multiplying by 4 tells you how many $\frac{1}{4}$'s are in a whole. Dividing by 3 adjusts this answer by accounting for the fact that it takes 3 of the $\frac{1}{4}$'s to make one object of the size you want.

Multiplying by the Reciprocal

The reciprocal approach may arise when working with a fraction divided by whole number. For example, with the expression $\frac{1}{2} \div 4$, students often draw the following diagram. A sample explanation is shown.



Here students are relating the expression $\frac{1}{2} \div 4$ to the expression $\frac{1}{2} \times \frac{1}{4}$. This type of reasoning, the diagram that develops it, and the number sentences that support it help students move from a division problem to multiplying by the reciprocal.

Common-Denominator Approach

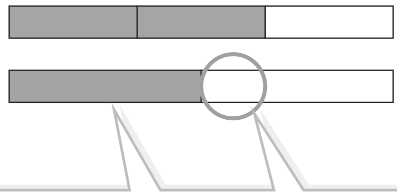
Some students intuitively try the same approach for division that worked in addition and subtraction—finding a common denominator. This algorithm nicely links to their whole-number understanding of division. A student explanation appears below.

> In the expression $\frac{7}{9} \div \frac{1}{3}$, I can rename the fractions. Then I can rewrite the expression, with a common denominator, as $\frac{7}{9} \div \frac{3}{9}$.
>
> So I have 7 one-ninth-size pieces and I want to find out how many groups of three one-ninth-size pieces I can make.
>
> I compute 7 ÷ 3, which equals $2\frac{1}{3}$.

This algorithm is used in *Decimal Ops* to develop decimal division.

The common denominator strategy for division of fractions can be modeled using fraction strips and the number line. It also can lead to the algebraic proof of the more traditional rule of "invert and multiply." Using a memorized rule is not in the spirit of CMP, but it is important that teachers understand these mathematical connections. A division problem presented in the introduction of Investigation 3 illustrates this connection.

Suppose you ask, "How many $\frac{1}{2}$'s are in $\frac{2}{3}$?" You can write the question as a division expression, $\frac{2}{3} \div \frac{1}{2}$. This division expression can also be represented by comparing fractions strips, as shown below.



To make $\frac{2}{3}$, you need a full $\frac{1}{2}$ and part of another $\frac{1}{2}$.

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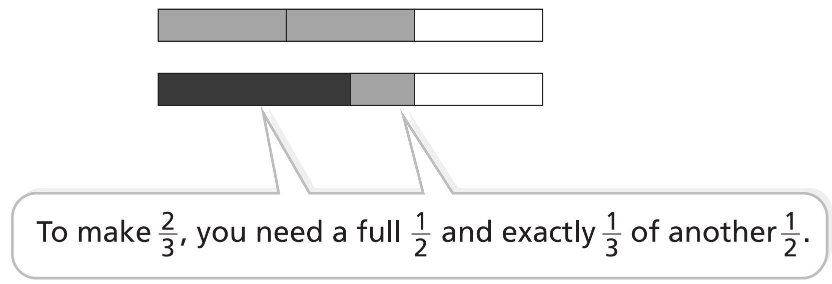
To completely solve this problem by comparing fraction strips, both strips in the figure below are divided into equal-sized pieces. This is essentially what the common denominator approach to solving a fraction division problem accomplishes.



The expression $\frac{2}{3}\left(\frac{2}{2}\right) \div \frac{1}{2}\left(\frac{3}{3}\right)$ is equivalent to $\frac{4}{6} \div \frac{3}{6}$, and since the pieces are now the same size, we can divide the numerators to get $\frac{4}{3}$, which can be written as the mixed numeral $1\frac{1}{3}$.

Note that in order to solve this problem by comparing fraction strips, we divided each strip into equal sized pieces. This is essentially what the common denominator approach to solving a fraction division problem accomplishes.

Relating Multiplication and Division

In additive situations, those involving addition and subtraction, the quantities are easy to count, measure, combine, and separate. This is because each quantity in an addition or subtraction problem has the same kind of label or is the same type of unit. For example, $3 + 4 = 7$ can be thought of as 3 marbles plus 4 marbles equals 7 marbles. Each quantity is a number of marbles.

In multiplicative situations, those involving multiplication and division, the quantities are not so straightforward. Each number may represent a different kind of unit. For example, if tomatoes cost 87 cents per can, the total cost for 6 cans can be found by multiplying 6 cans × 87 cents. It is hard to imagine a situation where adding tomatoes and money would make sense.

> **Example**
>
> **Units of measure do not change in addition.**
>
> marbles + marbles = marbles
>
> **Units of measure may change in multiplication.**
>
> cans × dollars per can = dollars

Another challenge is the different kinds of situations that call for multiplication and for division. A multiplication problem may be counting an array, or finding an area, or finding the sum of a repeated addition, and so on. Division may be finding how many groups of a certain size or measure that you can make from a given quantity or how many objects or parts would be in each of a given number of groups.

For example, the number sentence $3 \times 4 = 12$ could represent 3 people, each with 4 candies. The same number sentence could also represent 3 candies given to each of 4 people. The two types of division situations, *sharing* and *grouping*, are related to these two multiplication situations. The diagram below shows the grouping model of division first, followed by the sharing model.

Grouping

3	×	4	=	12	→	12	÷	4	=	3
Unit number of people		*Rate* candy per person		*Total* candies		*Total* candies		*Rate* candy per person		*Unit* number of people

Sharing

3	×	4	=	12	→	12	÷	4	=	3
Rate candy per person		*Unit* number of people		*Total* candies		*Total* candies		*Unit* number of people		*Rate* candy per person

It is important that students develop a sense of the kinds of situations for which each operation is useful. Therefore, you will see attention to meanings and interpretation of the operation in the unit.

Inverse Relationships

Fact families and missing-value problems are used to introduce the inverse relationships of addition and subtraction, and of multiplication and division. In elementary grades, students learn about fact families for whole numbers.

Whole-number fact family for the number sentence 3 + 5 = 8
$3 + 5 = 8$
$5 + 3 = 8$
$8 - 5 = 3$
$8 - 3 = 5$

continued on next page

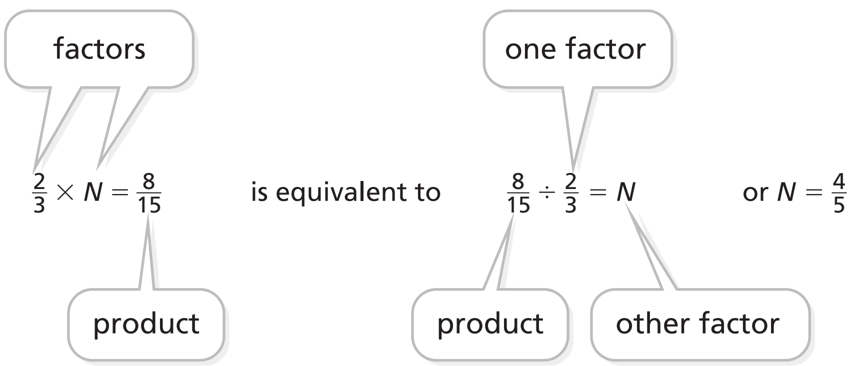
In *Let's Be Rational* these ideas are expanded to include fractions. Fact families can contain fractions as well as whole numbers.

Fraction fact family for the number sentence $\frac{7}{10} + \frac{1}{2} = \frac{6}{5}$
$\frac{7}{10} + \frac{1}{2} = \frac{6}{5}$
$\frac{1}{2} + \frac{7}{10} = \frac{6}{5}$
$\frac{6}{5} - \frac{1}{2} = \frac{7}{10}$
$\frac{6}{5} - \frac{7}{10} = \frac{1}{2}$

Understanding the inverse relationship between the operations pairs of addition-subtraction and multiplication-division is a tool that lends itself to many situations, one of which is solving equations. Do not expect students to develop formal procedures or notation for solving algebraic equations at this stage.

In *Let's Be Rational*, missing-value problems introduce students to the use of a variable as a placeholder. Such problems will help students begin to develop a generalized understanding of inverse relationships. In whole-number contexts, such as $20 \div N = 5$, students can solve for N by using multiplication and division facts with which they have experience. In a problem with fractions, such as $\frac{8}{15} \div N = \frac{2}{3}$, this becomes more difficult. Students have to think about which values are the factors in the related multiplication sentences $\frac{2}{3} \times N = \frac{8}{15}$ and $N \times \frac{2}{3} = \frac{8}{15}$.

Missing-Value Problems



Keep in mind that inverse relationships will be explored in other number contexts such as decimals and integers. Over time, students will start to think beyond the actual numbers to the relationships that exist among the values in fact families. This understanding will be a powerful tool for students to use in other mathematical contexts.