Mathematics Background

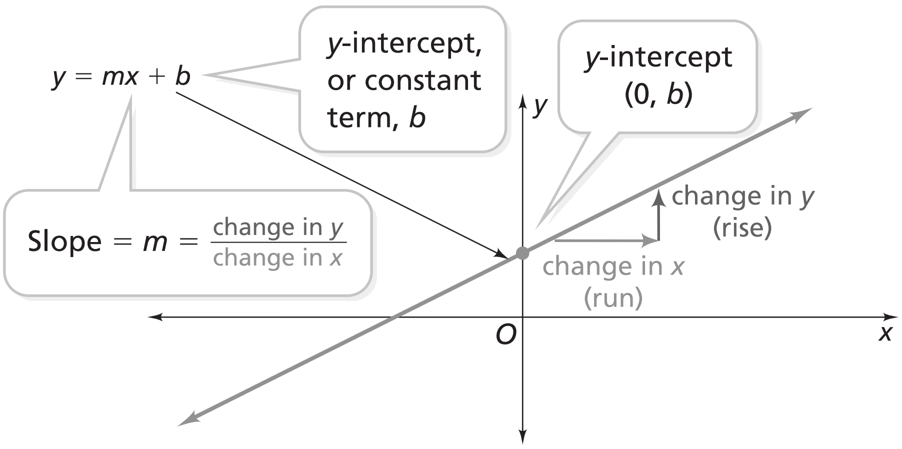
Linear Functions, Equations, and Inequalities

Although the basic understandings of and skills for linear equations were addressed in the Grade 7 Unit *Moving Straight Ahead*, they need to be revisited and practiced to deepen student understanding. The Problems in Investigation 2 of *Thinking With Mathematical Models* are designed to promote this sort of review and extension.

Linear Functions and Equations

In *Moving Straight Ahead*, students learned to recognize, represent symbolically, and analyze relationships in which a dependent variable changes at a constant rate relative to an independent variable.

Students learned the connections between the equation $y = mx + b$, the rate of change, the **slope** of the line, and the **y-intercept** of the line.



Solving Equations

Many questions about linear functions can be answered by solving equations of the form $c = mx + b$ for x (in which c is a constant).

In Moving Straight Ahead, students learned to approximate solutions to such equations by using tables and graphs of (x, y) values. They also learned to find exact solutions by reversing the operations to get $x = \frac{(c - b)}{m}$ and by using the *properties of equality*.

Properties of Equality	
Addition Property of Equality	
If you add the same number to each side of an equation, the two sides remain equal.	
Arithmetic	**Algebra**
$10 = 5(2)$, so $10 + 3 = 5(2) + 3$.	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	
If you subtract the same number from each side of an equation, the two sides remain equal.	
Arithmetic	**Algebra**
$10 = 5(2)$, so $10 - 3 = 5(2) - 3$.	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	
If you multiply each side of an equation by the same number, the two sides remain equal.	
Arithmetic	**Algebra**
$10 = 5(2)$, so $10 \cdot 3 = 5(2) \cdot 3$.	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	
If you divide each side of an equation by the same number, the two sides remain equal.	
Arithmetic	**Algebra**
$10 = 5(2)$, so $10 \div 3 = 5(2) \div 3$.	If $a = b$, then $a \div c = b \div c$.

Fact Families

Another way to solve equations is to use fact families.

Fact families with whole-number operations are introduced in Grade 6, and students revisit them in the Grade 7 Unit *Accentuate the Negative*. The concept of fact families highlights the relationships between addition and subtraction and between multiplication and division. Students can interpret subtraction problems as missing addend problems and division problems as missing factor problems.

To solve $c = mx + b$ for x, look at its fact family.

There are three equations in the addition and subtraction fact family for $c = mx + b$.

$c = mx + b$ $\qquad$ $c - b = mx$ $\qquad$ $c - mx = b$

To solve $c = mx + b$ for x, choose the equivalent equation $c - b = mx$.

There are three equations in the multiplication and division fact family for $c - b = mx$.

$c - b = mx$ $\qquad$ $\frac{(c - b)}{x} = m$ $\qquad$ $\frac{(c - b)}{m} = x$

To solve $c - b = mx$ for x, choose the equivalent equation $\frac{(c - b)}{m} = x$.
So the solution of $c = mx + b$ is $x = \frac{(c - b)}{m}$.

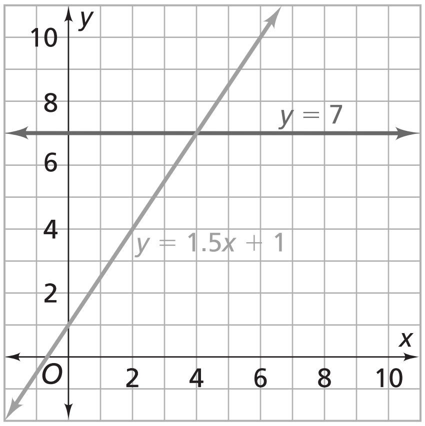
Inequalities

Many real-world problems involve inequalities rather than equations. Inequalities are mathematical sentences that use $\leq$, $\geq$, $<$, or $>$, such as $c \leq mx + b$ or $c \geq mx + b$.

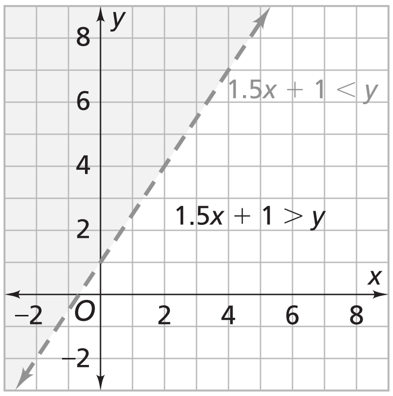
Problems in *Thinking With Mathematical Models* invite students to recognize the implications of phrases such as “at least” and “at most,” to use inequality notation for problem conditions, and to use tables and graphs for finding solutions of inequalities. Some inequalities have infinitely many solutions. Some inequalities, such as $x^2 \leq 0$, have one solution. Other inequalities, such as $x^2 < 0$, have no solution. The table shows that $1.5x + 1 > 7$ for $x > 4$.

x	y
−1	−0.5
0	1
1	2.5
2	4
3	5.5
4	7
5	8.5
⋮	⋮

The graph below shows another way of solving an inequality. The horizontal line $y = 7$ intersects the line $y = 1.5x + 1$ at the point (4, 7). Points on the line $y = 1.5x + 1$ above the horizontal line have y-values greater than 7 and x-values greater than 4. Thus, the solutions for $1.5x + 1 > 7$ are $x > 4$.



The graph below shows that the equation $y = 1.5x + 1$ divides the coordinate plane into two regions. In the shaded region above the line, $1.5x + 1 < y$ (or $y > 1.5x + 1$), and in the unshaded region below the line, $1.5x + 1 > y$ (or $y < 1.5x + 1$).



Properties of Inequality

The treatment of inequalities in this Unit is informal. Algebraic techniques for solving linear inequalities are covered in *It's in the System*.

The algebraic, numerical, and graphical strategies that lead to solutions of linear inequalities are related to those for equations, with some key differences. For example, you can multiply or divide both sides of an equation by a positive number without changing the solution. When you multiply or divide both sides of an inequality by a negative number, however, the direction of the inequality is reversed, as below.

This is a true number sentence.

$5 < 12$

Multiply both sides of the inequality by -1.

You do not get $-5 < -12$, which is a false number sentence.	Instead, you get $-5 > -12$, which is another true number sentence.

You may want to remind students of the properties of inequality.

Addition and Subtraction Properties of Inequality	
Addition Property of Inequality	
If you add the same number to each side of an inequality, the two sides remain equal.	
Arithmetic	**Algebra**
$8 < 12$, so $8 + 3 < 12 + 3$, and $10 > 7$, so $10 + 5 > 7 + 5$.	If $a < b$, then $a + c < b + c$, and if $a > b$, then $a + c > b + c$.
Note: These relationships are also true for $\leq$ and $\geq$.	
Subtraction Property of Inequality	
If you subtract the same number from each side of an inequality, the two sides remain equal.	
Arithmetic	**Algebra**
$8 < 12$, so $8 - 4 < 12 - 4$, and $10 > 7$, so $10 - 2 > 7 - 2$.	If $a < b$, then $a - c < b - c$, and if $a > b$, then $a - c > b - c$.
Note: These relationships are also true for $\leq$ and $\geq$.	

In particular, make sure students understand the difference between the multiplication and division properties of inequality with *positive* numbers and the properties with *negative* numbers.

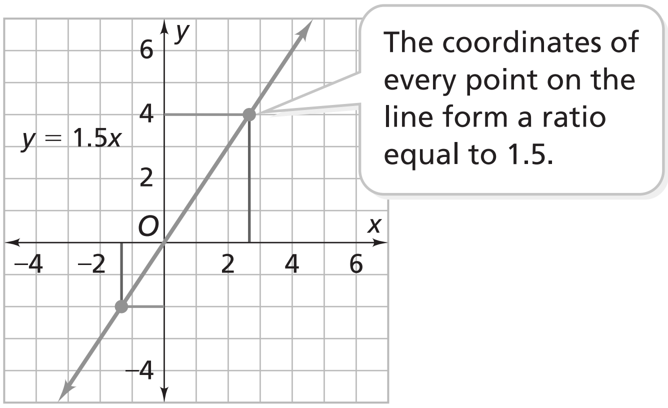
Multiplication and Division Properties of Inequality	
With *Positive* Numbers	
When you multiply or divide each side of an inequality by a *positive* number, the relationship between the sides does not change.	
Arithmetic	**Algebra**
$6 > 5$, so $6(3) > 5(3)$ and $\frac{6}{2} > \frac{5}{2}$.	If $a > b$ and $c > 0$, then $ac > bc$, and $\frac{a}{c} > \frac{b}{c}$.
$4 < 10$, so $4(5) < 10(5)$ and $\frac{4}{2} < \frac{10}{2}$.	If $a < b$ and $c > 0$, then $ac < bc$, and $\frac{a}{c} < \frac{b}{c}$.
Note: These relationships are also true for $\leq$ and $\geq$.	
With *Negative* Numbers	
When you multiply or divide each side of an inequality by a *negative* number, *reverse* the direction of the inequality sign.	
Arithmetic	**Algebra**
$6 > 5$, so $6 \cdot (-3) < 5 \cdot (-3)$ and $\frac{6}{-2} < \frac{5}{-2}$.	If $a > b$ and $c < 0$, then $ac < bc$, and $\frac{a}{c} < \frac{b}{c}$.
$4 < 10$, so $4 \cdot (-5) > 10 \cdot (-5)$ and $\frac{4}{-2} > \frac{10}{-2}$.	If $a < b$ and $c < 0$, then $ac > bc$, and $\frac{a}{c} > \frac{b}{c}$.
Note: These relationships are also true for $\leq$ and $\geq$.	

Direct Variation and Inverse Variation

Most people probably interpret the sentence "*y* varies directly with *x*" to mean "As *x* increases, *y* increases," and "*y* varies inversely with *x*" means "As *x* increases, *y* decreases." In mathematics, the meanings of direct variation and inverse variation are more specific.

Direct Variation

The sentence "*y* varies directly with *x*, or is directly proportional to *x*," means that there is some fixed number *k* such that $y = kx$. Below is a graph of the direct variation $y = 1.5x$.



The equation $y = kx$ implies that the ratio $\frac{y}{x}$ is equal to a constant value, *k*.

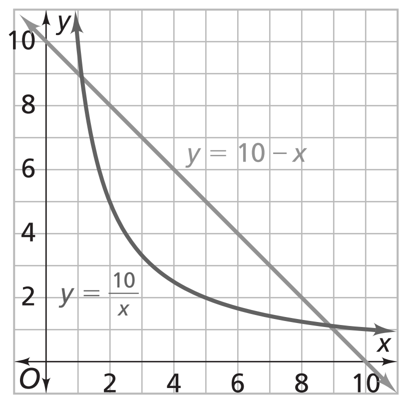
(This is why the word *proportional* is used in the preceding statement of direct variation.)

So, for example, when *x* is doubled, *y* doubles, and when *x* is tripled, *y* triples.

Note: The proportionality of *x* and *y* is not true for linear relationships of the form $y = kx + b$ for which $b \neq 0$. The special case of $b = 0$ is a direct variation.

Inverse Variation

Most people probably interpret an inverse variation as simply a relationship in which *y* decreases as *x* increases. For example, the graph below shows a line and a curve. For both, the value of *y* decreases as *x* increases. In mathematics, however, $y = \frac{10}{x}$ is called an inverse variation, and $y = 10 - x$ is not.



An inverse variation can also be written in the form $xy = k$, which emphasizes that the product of the two variables is constant. In an inverse variation, multiplying x by n multiplies y by $\frac{1}{n}$. So, for example, doubling x halves y, and tripling x multiplies y by $\frac{1}{3}$.

Direct and Inverse Variations With Formulas

Investigation 3 uses several familiar contexts to develop the concept of inverse variation, building on students' experiences with formulas such as $A = \ell w$ and $d = rt$.

The formula for the area of a rectangle was first explored in the Grade 6 Unit *Covering and Surrounding*. In *Thinking With Mathematical Models*, rather than finding the area of a rectangle with a given length and width, as they did in Grade 6, students look for combinations of length and width values that give a fixed area.

Use to calculate combinations of ℓ and w efficiently.

This leads to the formula $\ell = \frac{A}{w}$.

Students also explore the formula $d = rt$, which relates distance, rate, and time. In earlier Units, students calculated the distance traveled for a given rate and time (a direct variation). In this Unit, they find combinations of rate and time values that give a fixed distance.

Use to calculate combinations of r and t efficiently.

This leads to inverse variation functions of the form $r = \frac{d}{t}$ and $t = \frac{d}{r}$.

The Grade 6 Unit *Prime Time* covered the relationship between factor pairs of a number and rectangles with area equal to the number. By superimposing the factor-pair rectangles for a number on top of each other, students could see the symmetry of the factor pairs. Graphing those factor pairs as coordinates shows an inverse variation relationship. Visit *Teacher Place* at mathdashboard.com/cmp3 to see the complete video.

Mathematical Modeling

A key idea of *Thinking With Mathematical Models* is to use mathematics to approximate real-world data. Mathematics provides precisely defined objects and operations that can be used to represent real-world data patterns that may not be as well behaved.

Effective use of mathematical modeling requires awareness of the overall modeling process and a set of mathematical concepts and skills for model building and analysis. This Unit is only an introduction to these ideas. It lays a foundation for a more sophisticated and thorough development of modeling strategies in high school and college mathematics and science courses.

Modeling in Thinking With Mathematical Models

In this Unit, students learn about situations in which linear or inverse variation models are particularly appropriate. For the linear examples, students "eyeball" a fitted line and then find the equation of the line. Students do not find the line of best fit for the data.

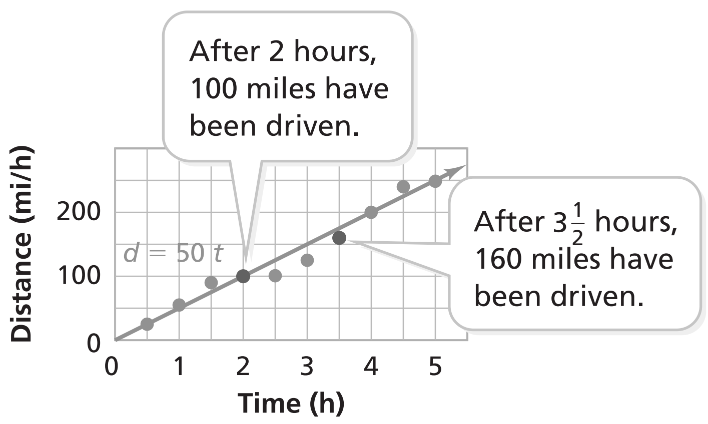
For the inverse variation examples, we suggest that students only experiment with data plots and test function rules to establish reasonable proportionality constants.

Using the plotting and function-graphing capabilities of a graphing calculator makes successive approximation an effective modeling technique. If you have a graphing calculator available, you may use it to extend student investigation for any of the Problems in this Unit.

Using Models to Approximate Data

Suppose you used the equation $d = 50t$ to model the distance traveled as a function of elapsed time. You could make fairly good general predictions using the model, but the equation would probably not give the exact distance for a specific time, because it is unlikely that a constant speed can be maintained throughout.

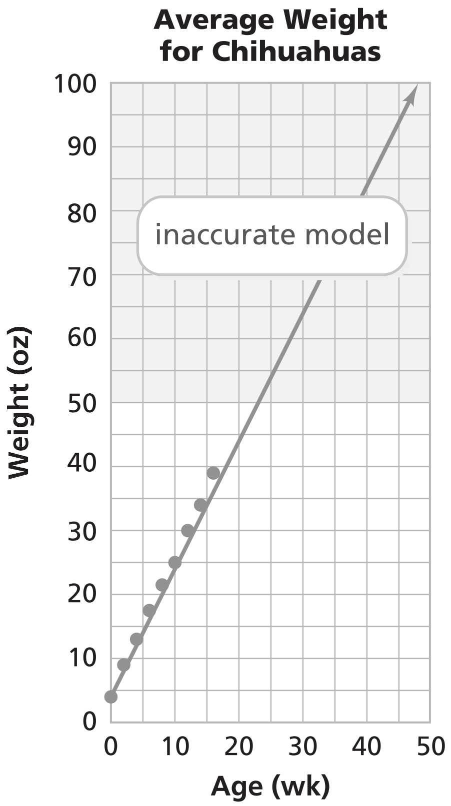
In our fictitious example below, some data points are below the model, some are on it, and some are above it.



Reality Check

It is important to realize that models approximate data and may be useful only for a certain range of values.

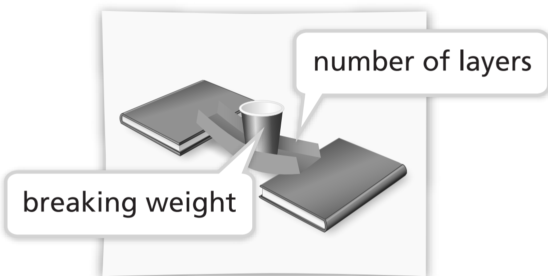
For example, in one ACE exercise, students model the relationship between the age and weight of a Chihuahua based on data for the first few months of a Chihuahua's life. They find that their model is useful for only a limited number of weeks because, after a certain age, dogs stop growing.



Steps for Using Models

Using mathematical modeling to solve quantitative problems involves at least five basic steps.

Step 1 Identify the key variables involved in the problem situation.

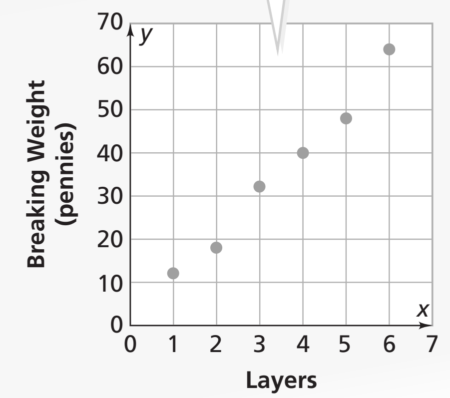


Step 2 Collect data that indicate the nature of the relationship between the variables.

student data

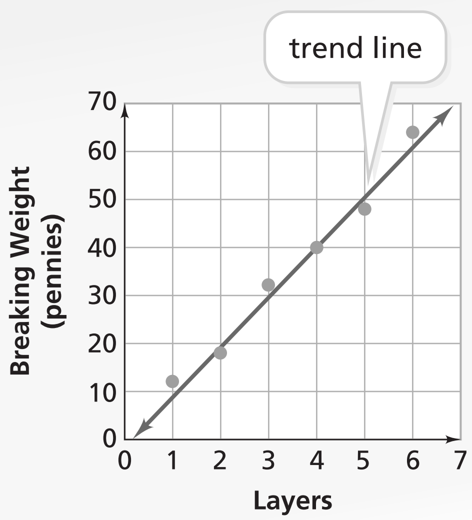
Bridge Thickness (layers)	Breaking Weight (pennies)
1	12
2	18
3	32
4	40
5	48
6	64

Problem **2.1**



Problem **2.1**

Step 3 Find an algebraic equation that approximates, or models, the relationship.



Problem **2.1**

Step 4 Use the model to write and solve equations or to make calculations that provide information about values between or beyond the data values.

> The linear function that models the data is y = 10x. A bridge made from five layers of paper should have a breaking weight of about 50 pennies.

Step 5 Interpret the results of the mathematical calculations in the context of the original problem.

mathematical model	what the model means	
y = 10x	breaking weight of the bridge	= 10 pennies x number of layers

Variability in Data

Understanding the Concept of Distribution

When students work with data, they are often interested in the individual data items, particularly if the data are about themselves. Looking at the overall distribution of a data set rather than at individual items can reveal important information.

We use graphs to help provide a picture of a distribution of data. Distributions (unlike individual cases) have properties that include statistics such as measures of central tendency (i.e., mean, median, mode) or variability (e.g., outliers, range) and characteristics such as shape (e.g., clumps, gaps, skewed distributions).

Variability and Why It Is Important

When we look at distributions, we are often interested in the measures of center, which tell us that a value is "typical" of the distribution. Any measure of center alone can be misleading, however. It is important also to consider the *variability* of the distribution.

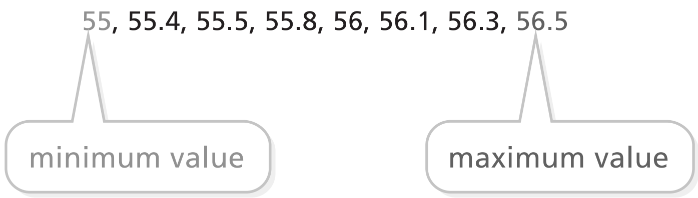
Generally, students' earlier work with data analysis has emphasized describing what is typical about a distribution of data. During the middle grades, there is a shift toward consideration of variability; students are better prepared mathematically and developmentally to consider this concept.

Describing variability includes looking at measures of center, at the range of the data, at where data cluster or where there are gaps in a distribution, at the presence of outliers, and at the shape of the distribution.

Using Measures of Variability

Measures of variability establish the degree of spread of the individual data values and their deviations (or differences) from the measures of center.

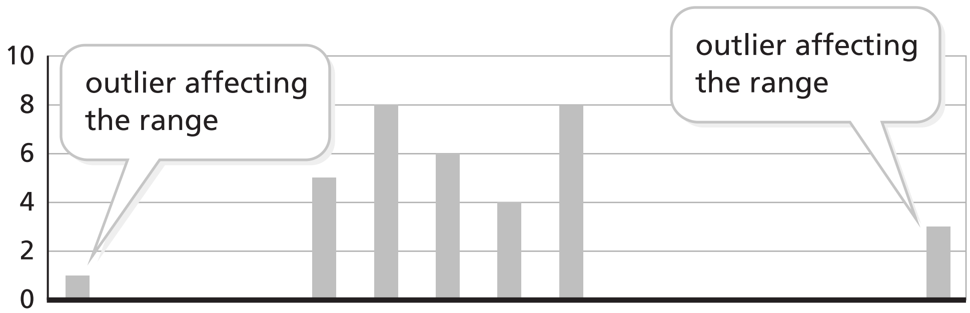
In CMP3 data units, we use **minimum value** and **maximum value** to specify the least value and the greatest value of a data set.



The **range** is a number found by subtracting the minimum value from the maximum value.

range = maximum value − minimum value
1.5 = 56.5 − 55

In some cases, data will be uniformly distributed between maximum and minimum values. At other times, most of the data will be clustered, with outliers, as in the diagram below.



Standard Deviation and Mean Absolute Deviation (MAD)

The concept of a mathematical model is based on the fact that few real phenomena are as well behaved as the functions of formal mathematics. In almost every situation, experimental data will only approximate linear, quadratic, exponential, or inverse variation patterns. Thus an important aspect of constructing and using mathematical models is dealing with the inevitable variations in real-world data.

Statisticians typically measure the spread of a distribution by using its **variance** and its **standard deviation**. Roughly speaking, the variance is the average of the squared differences between data values and the mean, and the standard deviation is the square root of the variance.

Calculating standard deviation is similar to calculating **mean absolute deviation** (MAD), which students have done in earlier grades. Statisticians prefer the standard deviation over MAD, because it is easier to calculate and has been incorporated into many statistical formulas used in analysis.

Follow these steps to calculate standard deviation.

Step 1 Find the mean of the data set.

Step 2 Calculate the difference of each data point and the mean.

Step 3 Square the differences.

Squaring the differences makes them all positive, as MAD does, but also increases their weight when the differences are greater than 1.

Step 4 Divide the sum of the squared differences by a number 1 less than the number of data points.

This number is called the *variance*.

Step 5 Take the square root of the variance.

This brings the average of the differences back to the same scale as the original data, which compensates for the squaring of the differences.

Standard Deviation Formula

When you analyze data of a sample from a population, you can get unbiased estimates of the standard deviation in a somewhat nonintuitive calculation. You do not, as you might expect, find the square root of the mean of the squared deviations from the mean. Instead, you divide the sum of squared deviations by 1 less than the number of deviations.

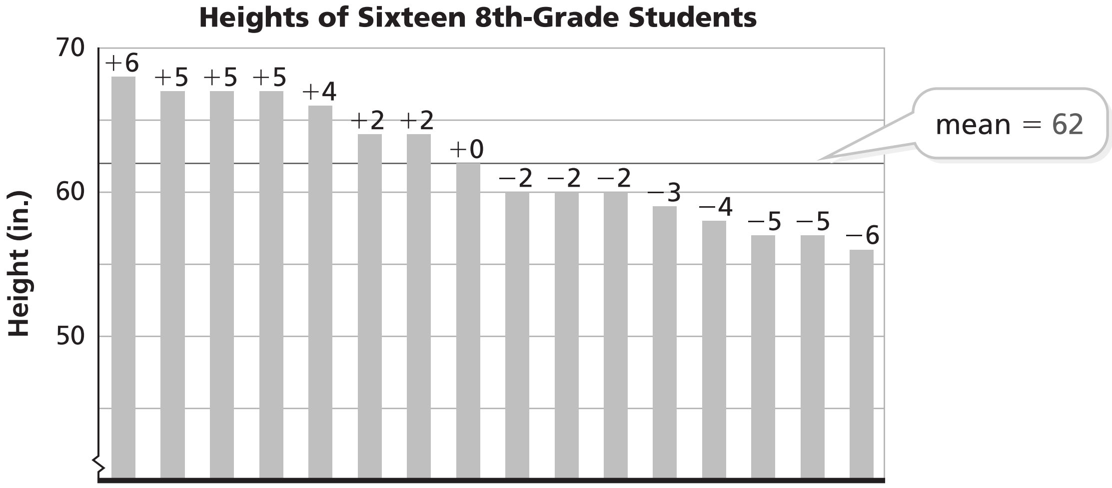
That is, the standard deviation of a sample is given by the formula below.

$$\sigma_x = \sqrt{\frac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n-1}}$$

For a reasonably large data set (value of *n*), the difference between dividing by *n* and dividing by $n - 1$ is generally insignificant. Calculators often give both the sample standard deviation s_x and the population standard deviation σ_x for a set of data. So if students use a calculator or spreadsheet to find standard deviation, they might come up with numbers that are slightly different from what we offer in the answers to problems and ACE exercises.

Comparing Standard Deviation and MAD

The bar graph below shows the heights of sixteen students. It also shows the differences from the mean of those students. The tables that follow compare the standard deviation of the data with its mean absolute deviation.



The tables below show two calculations from the data above.

Computing MAD	
Difference	**Absolute Value of the Difference**
68 − 62 = 6	6
67 − 62 = 5	5
67 − 62 = 5	5
67 − 62 = 5	5
66 − 62 = 4	4
64 − 62 = 2	2
64 − 62 = 2	2
62 − 62 = 0	0
60 − 62 = −2	2
60 − 62 = −2	2
60 − 62 = −2	2
59 − 62 = −3	3
58 − 62 = −4	4
57 − 62 = −5	5
57 − 62 = −5	5
56 − 62 = −6	6
Sum of absolute values	58
MAD Divide the sum by *n*.	3.625

Computing Standard Deviation	
Difference	**Squared Difference**
68 − 62 = 6	36
67 − 62 = 5	25
67 − 62 = 5	25
67 − 62 = 5	25
66 − 62 = 4	16
64 − 62 = 2	4
64 − 62 = 2	4
62 − 62 = 0	0
60 − 62 = −2	4
60 − 62 = −2	4
60 − 62 = −2	4
59 − 62 = −3	9
58 − 62 = −4	16
57 − 62 = −5	25
57 − 62 = −5	25
56 − 62 = −6	36
Sum of squared differences	258
Variance Divide the sum of squared differences by *n* − 1.	17.2
Standard deviation Take the square root of variance.	4.15

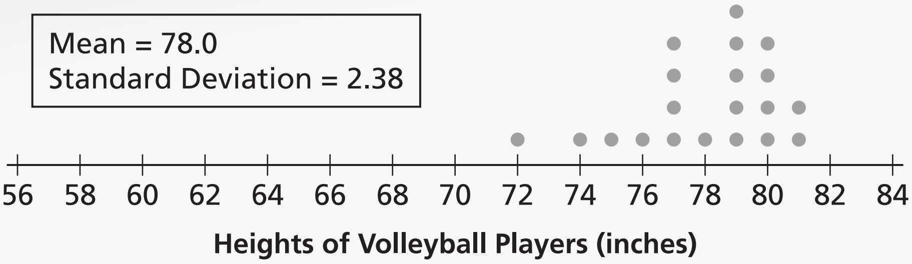
Using Standard Deviation

As we mentioned above, standard deviation is a measure of spread. Suppose you have two data sets with the same mean. They will not necessarily have the same standard deviation. In fact, the data set with the greater spread will have a greater standard deviation.

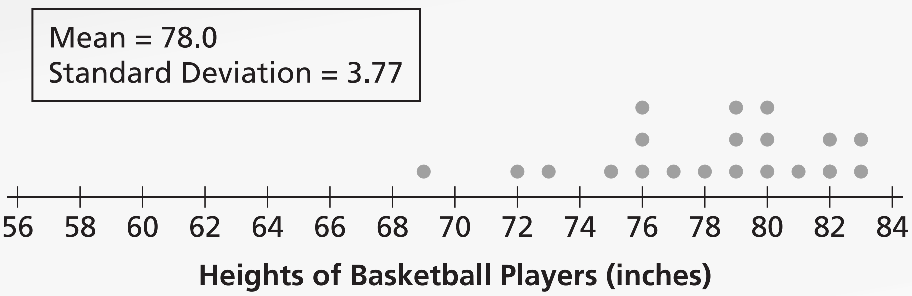
Example

The two line plots below show sports teams with the same mean height but different standard deviations. The standard deviations tell us that the heights of the players on the basketball team are more widely varied than the heights of the players on the volleyball team.

Note that the standard deviations do not tell us *why* the heights of the basketball players are more varied, only that they are more varied.



Problem **4.4**



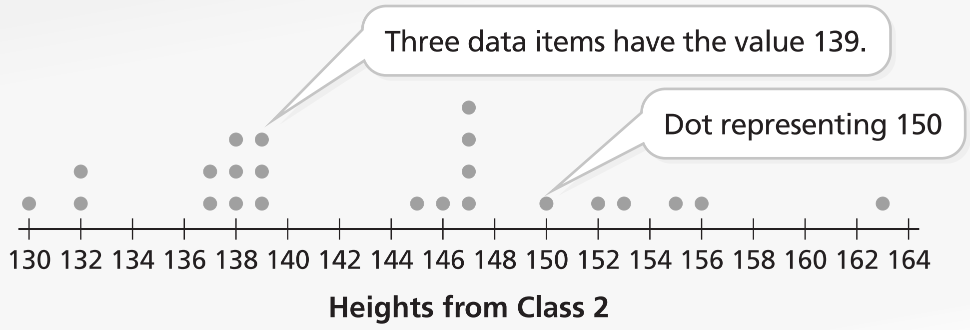
Problem **4.4**

Graphing Statistical data

In *Thinking With Mathematical Models*, we use several different types of graphs to display data.

Line Plot

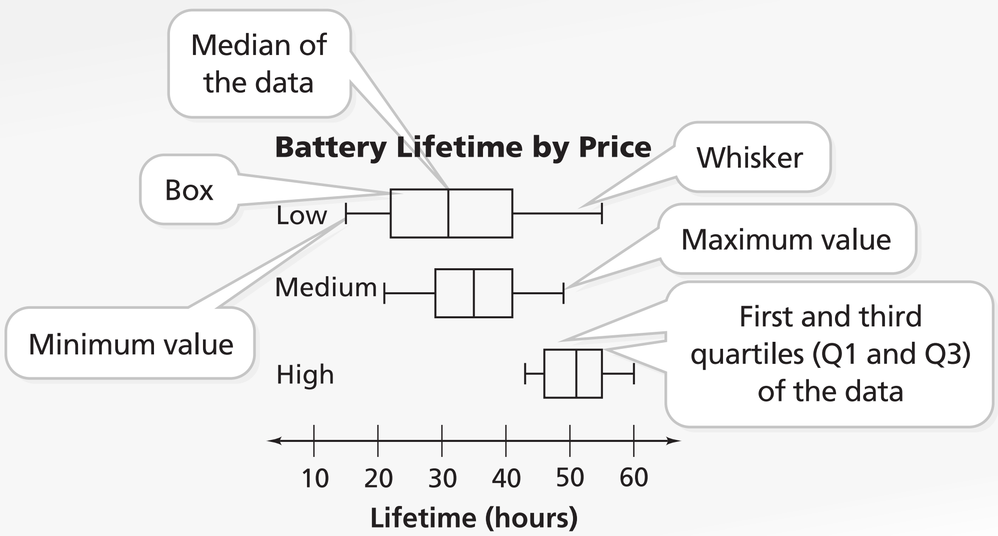
In a **line plot**, each piece of data is represented by a dot (or another mark, such as an X) positioned over a number line.



Investigation **4** ACE

Box-and-whisker plot

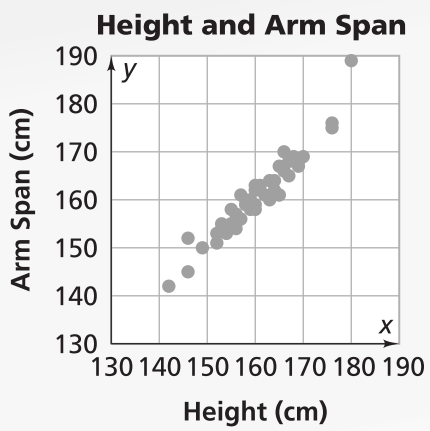
A **box-and-whisker plot** is divided into quartiles and displays properties of distributions, such as symmetry or skewness. This type of graph was developed largely because comparing data using frequency bar graphs can be confusing, especially if one is comparing more than two bar graphs.



Investigation **5** ACE

Scatter plot

A **scatter plot** explores the relationship between two variables. When each subject in a statistical study is assigned two numerical measures, the results can be represented as ordered pairs of numbers and displayed as points on a coordinate graph.



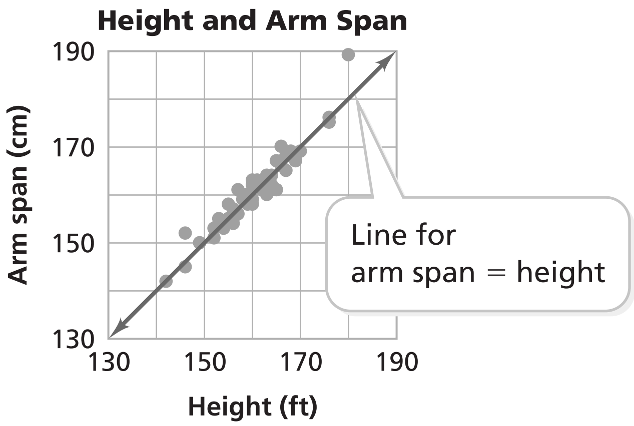
Investigation **4** ACE

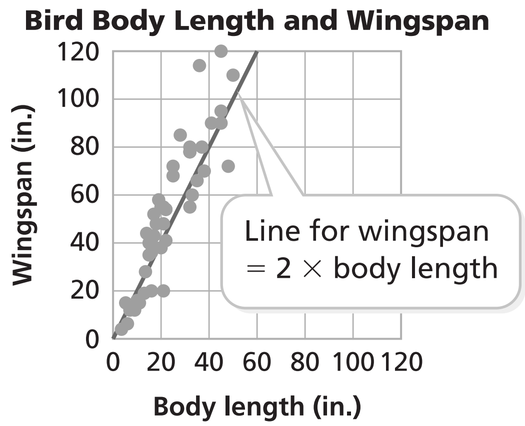
Exploring the Concept of Covariation, or Association

When the behavior of the values of two different attributes is related in a meaningful way, then information about values from one attribute can help us predict values of the other attribute. This does not necessarily imply that change in one variable causes change in the other variable. Ideas such as fitting a line to and characterizing the strength of a relationship between paired data values for two attributes emerge as ways of describing how the data are distributed. Fitting a line may be explored informally using a basic understanding of linearity.

During the Investigations in *Thinking With Mathematical Models*, students develop an awareness of how the covariance of attributes of a data situation can be seen in a scatter plot.

Some of the relationships explored involve proportional relationships (e.g., height and arm span for people, body length and wingspan for birds); equations for lines characterizing these relationships have a y-intercept of 0.

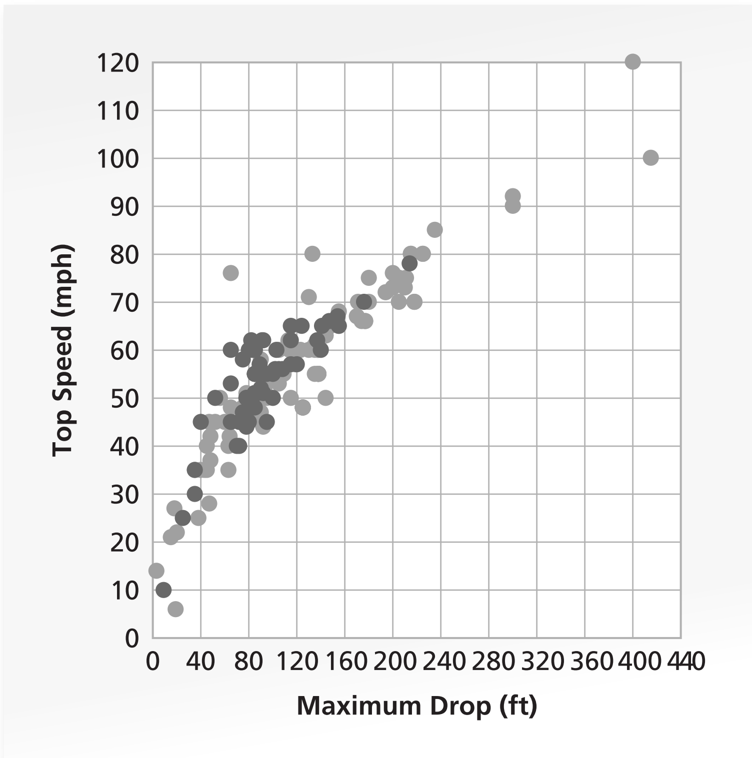




Continuing to Explore the Concept of Covariation

Covariation is a way of characterizing a relationship between two (most often) numerical attributes. It means that information about values from one attribute helps us predict values of the other attribute. Students' work with covariation is informal and very concrete.

In *Thinking With Mathematical Models*, students consider whether knowing one attribute might help them understand the variability in another attribute. For instance, is the top speed of a roller coaster related to the maximum drop along the track? The graph shows that as the length of the maximum drop increases, so does the top speed of the coaster.



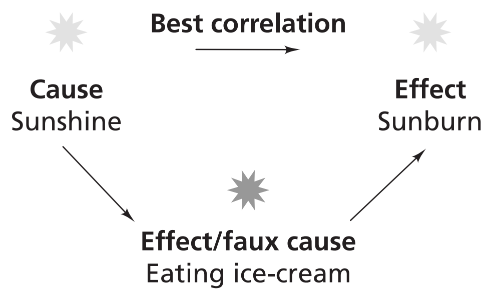
Correlation Coefficient

The **correlation coefficient** is a number between 1 and −1 that tells how closely a pattern of data points fits a straight line. Visit *Teacher Place* at mathdashboard.com/cmp3 to see the complete video.

At this point we are not aiming to develop the correlation coefficient formula. Most statistical calculators and computer software programs produce correlation coefficient numbers.

One of the most important ideas for students to understand is that a correlation does not prove a cause-and-effect relationship between two variables.

For example, sales of ice cream are correlated with incidence of sunburn (both are higher in the summer and lower in the winter), but ice cream cones don't cause sunburn. This sort of informal introduction to correlation is a Common Core State Standard for Grade 8.



Categorical Variables and Two-Way Tables

Many problem-solving and decision-making situations require analyzing what statisticians call *categorical variables*. For example, as the student text points out, in comparing the popularity of wood-frame and steel-frame roller coasters, the type of roller coaster frame is a categorical variable that has values *wood* and *steel*.

Data may fall into more than two categories. For instance, a study comparing the popularity of cars might investigate the categorical variable *type of vehicle* with values *sedan*, *convertible*, *SUV*, *pickup*, and so on. A study comparing the popularity of dogs could investigate the categorical variable *breed of dog* with values *poodle*, *terrier*, *Irish setter*, *German shepherd*, *Chihuahua*, and so on.

Questions about categorical variables are often resolved by comparing frequencies of occurrence for each categorical value. The Common Core standards for Grade 8 call for an introduction to associations between pairs of categorical variables by analyzing two-way tables.

Analyzing Two-way Tables

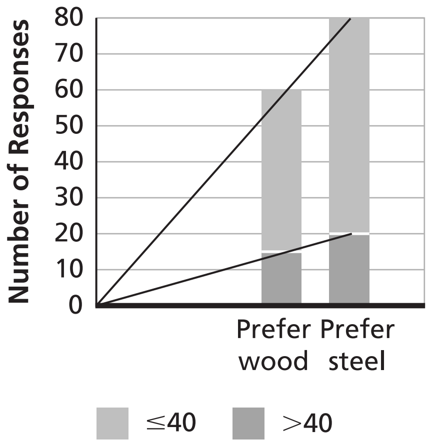
In Investigation 5 of *Thinking With Mathematical Models*, we ask students what data in the following table show about preferences of older and younger riders for wood-frame and steel-frame roller coasters.

	Prefer Wood	Prefer Steel	Prefer Steel
Age ≤ 40 years	45	60	105
Age > 40 years	15	20	35
Total	60	80	140

A first glance at the data might suggest that young people like wooden coasters more than older people do. The marginal totals in the table, however, call for more careful reasoning.

In fact, the same proportions of each group (younger and older riders) prefer wood-frame and steel-frame coasters. The data that provide this insight are not the raw survey numbers, but the fractions or percents of the appropriate totals in the sample.

The two lines drawn from the origin on the bar graph below show the proportionality of the responses.



Reality Check

Actual data sets are rarely perfectly proportional. This means that statistical analysis of questions such as “Is there a relationship between age and preference of roller coaster type?” must include an analysis of the proportions in each table cell. When analyzing actual data, statisticians must consider what defines a significant difference. The data sets in CMP, however, do not require attention to that issue.

Reading Standard Graphs

As a central component of data analysis, graphs deserve special attention. In a study of graph comprehension to assess the understanding of students in Grades 4 and 7 of four traditional graphs (pictographs, bar graphs, circle graphs, and line graphs), three components of graph comprehension were identified that are useful here.

- *Reading the data* involves lifting information from a graph to answer explicit questions.
- *Reading between the data* includes the interpretation and integration of information presented in a graph.
- *Reading beyond the data* involves extending, predicting, or inferring from data to answer implicit questions. Reading beyond the data helps students to develop higher-level thinking skills, such as inference and justification.

Using measures of central tendency or location

The three measures of central tendency—mode, mean, median—were addressed in *Data About Us*. In *Thinking With Mathematical Models*, understanding and fluency in the use of these measures is assumed.

mean A value that represents the "evening out" of the values in a set of data. If all the data had the same value, the mean would be that value.

median The numerical value that marks the middle of an ordered set of data. Half the data occur above the median, and half the data occur below the median.

mode The category or numerical value that occurs most often. It is possible for a set of data to have more than one mode.

Percents and Circle Graphs

In *Thinking With Mathematical Models*, percents are used to draw circle graphs correctly. When drawing a circle graph, students need to find the number of degrees from relative frequencies.

To draw a circle graph representing a collection of cubes in which 3 are blue, 6 are red, and 5 are yellow, you need to calculate the number of degrees for each section.

There are 14 cubes in all. Use fractions to represent the fractional part of the data each color represents. Then calculate the corresponding percents.

$\frac{3}{14}$ blue; $\frac{5}{14}$ yellow; $\frac{6}{14}$ red

Use the equivalent percents to multiply by 360°.

$\frac{3}{14} \approx 21\%$ and $0.21 \times 360° = 75.6°$

$\frac{5}{14} \approx 36\%$ and $0.36 \times 360° = 129.6°$

$\frac{6}{14} \approx 43\%$ and $0.43 \times 360° = 154.8°$

Draw the circle graph.