Mathematics Background

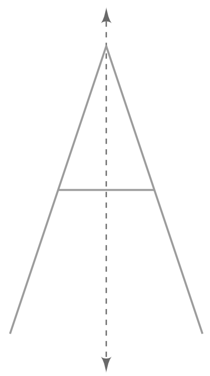
Types of Symmetry

In this Unit, students study symmetry and transformations. They connect these concepts to congruence and similarity. Symmetry and transformations have actually been studied in the Grade 7 Unit *Stretching and Shrinking*. In this Unit, students learn to recognize and make designs with symmetry, and to describe mathematically the transformations that lead to symmetric designs. They explore the concept and consequences of congruence of two figures by looking for symmetry transformations that will map one figure exactly onto the other.

In the first Investigation, students learn to recognize designs with symmetry and to identify lines of symmetry, centers and angles of rotation, and directions and lengths of translations.

Reflections

A design has reflection symmetry, also called mirror symmetry, if a reflection in a line maps the figure exactly onto itself. For example, the letter A has reflection symmetry because a reflection in a vertical line will match each point on the left half with a point on the right half. The vertical line is the line of symmetry for this design.



Rotations

A design has rotation symmetry if a rotation, other than a full turn, about a point maps the figure onto itself. The design below has rotation symmetry because a rotation of 120° or 240° about point *P* will match each flag to another flag. Point *P* is called the center of rotation. The angle of rotation for this design is 120°, which is the smallest angle through which the design can be rotated to match with its original position.

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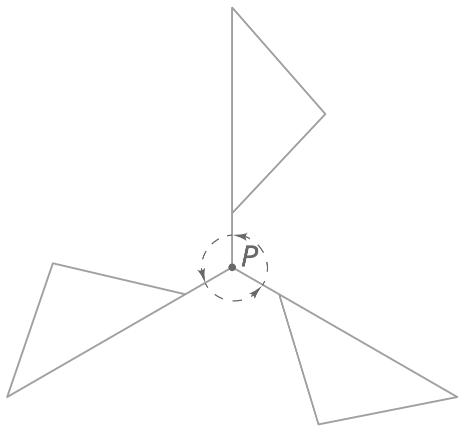
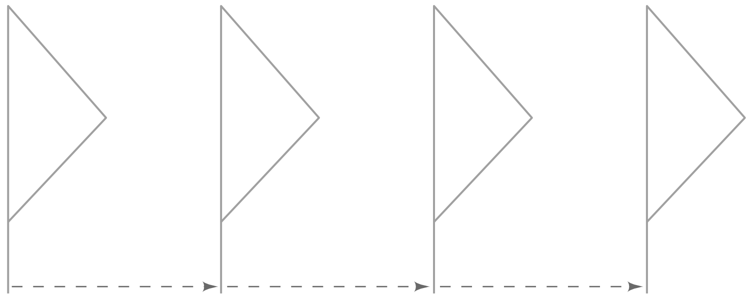


Translations

A design has translation symmetry if a translation, or a slide, maps the figure onto itself. The figure below is part of a translation-symmetric design. If this design continued in both directions, a slide of 1 inch to the right or left would match each element in the design with an identical copy of that design element.



Making Symmetric Designs

Once students learn to recognize symmetry in given designs, they can make their own symmetric designs. Students may use reflecting devices, tracing paper, angle rulers or protractors, and geometry software to help them construct designs.

- A design with reflection symmetry can be made by starting with a basic figure and then drawing the reflection of the figure in a line. The original and its reflection image make a design with reflection symmetry.
- A design with rotation symmetry can be made by starting with a basic figure and making $n - 1$ copies of the figure, where each copy is rotated $\frac{360°}{n}$ about a center point starting from the previous copy. The original and its $n - 1$ rotation images make a design that has rotation symmetry.
- A figure with translation symmetry can be made by copying the basic figure, so that each copy is the same distance and same direction from the previous copy. The figure and its translation images make a design with translation symmetry.

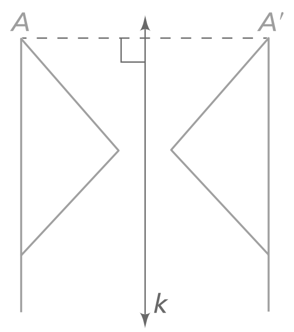
Students are asked to develop two separate but related skills. The first is to recognize symmetries within a given design. The second is to make designs with one or more specified symmetries starting with an original figure (which may not, in itself, have any symmetries). Thus, it is important to give students experience both in analyzing existing designs to identify their symmetries and also in using transformations to make designs that have symmetry.

Symmetry Transformations

The concepts of symmetry are used as the starting point for the study of symmetry transformations, also called distance-preserving transformations, rigid motions, or isometries. The most familiar distance-preserving transformations—reflections, rotations, and translations—"move" points to image points so that the distance between any two original points is equal to the distance between their images. The informal language used to specify these transformations is *slides*, *flips*, and *turns*. Some children will have used this language and will have had informal experiences with these transformations in the elementary grades.

Reflections

In this Unit, students examine figures and their images under reflections, rotations, and translations by measuring key distances and angles. They use their findings to determine how they can specify a particular transformation so that another person could perform it exactly. Students learn that a reflection can be specified by its line of reflection. They learn that, under a reflection in a line k, the point A and its image point A' lie at opposite ends of a line segment that is bisected at right angles by the line of reflection.



Rotations

A rotation can be specified by giving the center of rotation and the angle of the turn. In this Unit, the direction of the rotation is assumed to be counterclockwise unless a clockwise turn is specified. For example, a 57° rotation about a point C is a counterclockwise turn of 57° with C as the center of the rotation. Students learn that a point R and its image point R' are equidistant from the center of the rotation C.

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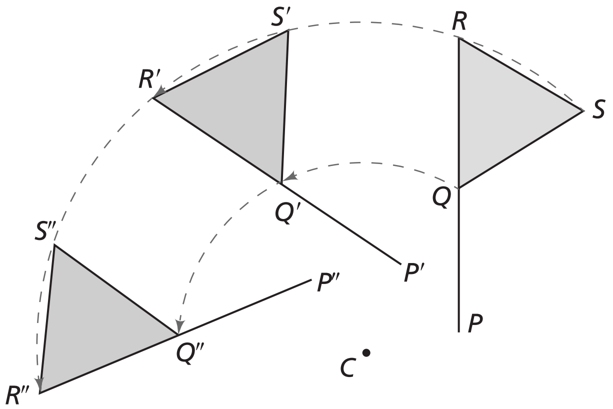
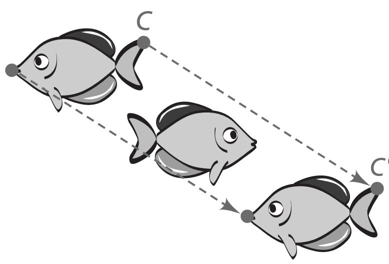
They see that a point under a rotation travels on the arc of a circle and that the set of circles on which the points of the figure travel are concentric circles with center *C*. They also find that the angles formed by the vertex points of the figure and their rotation images, such as $\angle RCR'$, all have a measure equal to the angle of turn.



Translations

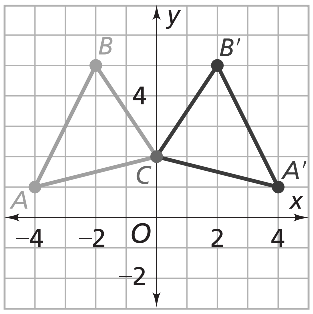
A translation can be specified by giving the length and direction of the slide. This can be done by drawing an arrow with the appropriate length and direction. Students find that if you draw the segments connecting points to their images, such as $\overline{CC'}$, the segments will be parallel and all the same length. The length is equal to the magnitude of the translation.



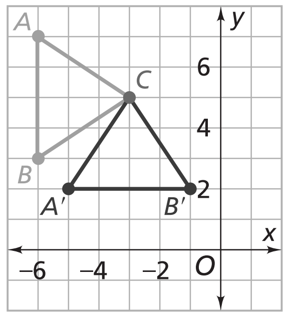
This work helps students to realize that any transformation of a figure is essentially a transformation of the entire plane. For every point in a plane, a transformation locates an image point. It is not uncommon to focus on the effect of a transformation on a particular figure. This Unit attempts to give mathematically precise descriptions of transformations while accommodating students' natural instinct to visualize the figures moving. Thus, in many cases, students are asked to study a figure and its image without considering the effect of the transformation on other points. However, the *moved* figure is always referred to as the image of the original, and the vertices of the image are often labeled with primes or double primes to indicate that they are indeed different points.

An interesting question is, "For which transformations are there points that remain fixed?" These are called *fixed points*. The image of each such point is simply the point itself. For a reflection, the points on the line of reflection are fixed points. For a rotation, the only fixed point is the center of rotation. For a translation, all points have images with new locations, so there are no fixed points. Point *C* is a fixed point in the reflection and rotation below.

Reflection



Rotation



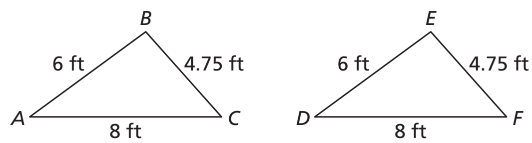
Congruent Figures

The discussion of distance-preserving transformations leads naturally to the idea of congruence. Two figures are congruent if they have the same size and shape. Intuitively, this means that you could "move" one figure exactly onto the other by a combination of symmetry transformations (rigid motions). In the language of transformations, two figures are congruent if there is a combination of distance-preserving transformations (symmetry transformations) that maps one figure onto the other. Several problems ask students to explore this fundamental relationship among geometric figures.

The question of *proving* whether two figures are congruent is explored informally. An important question is what minimum set of equal measures of corresponding sides and/or angles will guarantee that two triangles are congruent. It is likely that students will discover the following triangle congruence theorems that are usually taught and proved in high school geometry. This engagement with the ideas in an informal way will help make their experience with proof in high school geometry more understandable.

- Side-Side-Side

If the three sides of one triangle are congruent to three corresponding sides of another triangle, the triangles will be congruent (in all parts).



This condition is commonly known as the Side-Side-Side or SSS Postulate.
In the diagram above, $\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, and $\overline{AC} = \overline{DF}$.
So $\triangle ABC \cong \triangle DEF$ by the SSS Postulate.

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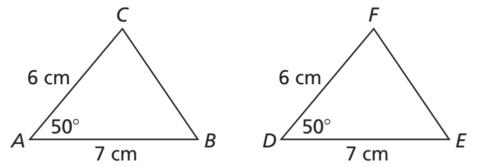
- Side-Angle-Side

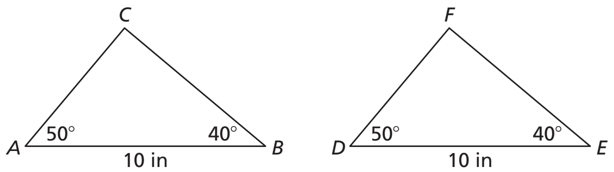
If two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of another triangle, the triangles will be congruent (in all parts).



This condition is commonly known as the Side-Angle-Side or SAS Postulate.
In the diagram above, $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$.
So $\triangle ABC \cong \triangle DEF$ by the SAS Postulate.

- Angle-Side-Angle

If two angles and the included side of one triangle are congruent respectively to two angles and the included side of another triangle, the triangles will be congruent (in all parts).



This condition is commonly known as the Angle-Side-Angle or ASA Postulate.
In the diagram above, $\angle A \cong \angle D$, $\overline{AB} = \overline{DE}$, and $\angle B \cong \angle E$.
So $\triangle ABC \cong \triangle DEF$ by the ASA Postulate.

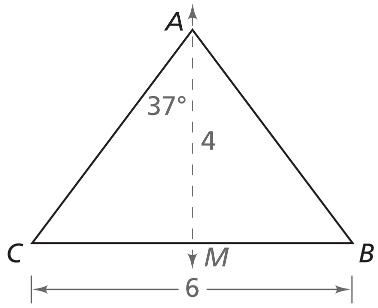
Students should also find that Angle-Angle-Angle and Side-Side-Angle do not guarantee congruence. Angle-Angle-Angle guarantees similarity, or the same shape, but not the same size. With Side-Side-Angle, in some cases there are two possibilities, so you cannot know for certain that you have congruence.

In a right triangle, with the right angle and any two corresponding sides given, you can use the Pythagorean Theorem to find the third side. This gives two sides and the included angle, or three sides. Either combination is enough to know that two right triangles are congruent.

Reasoning From Symmetry and Congruence

Symmetry and congruence give us ways of reasoning about figures that allow us to draw conclusions about relationships of line segments and angles within the figures.

For example, suppose that $\overleftrightarrow{AM}$ is a line of reflection symmetry for triangle *ABC*; the measure of $\angle CAM$ is 37°; the length of $\overline{CB} = 6$; and the length of $\overline{AM} = 4$.

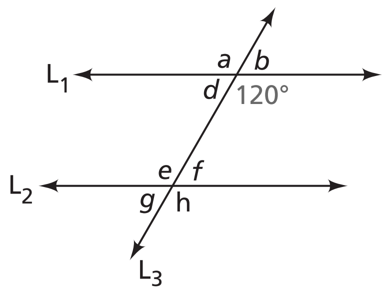


As a consequence of the line symmetry, you can say that

- Point *C* is a reflection of point *B*.
- Point *A* is the reflection of point *A*.
- Point *M* is the reflection of point *M*.
- $\overline{AC}$ is a reflection of $\overline{AB}$, which means that their lengths are equal.
- $\overline{CM}$ is a reflection of $\overline{BM}$, so each has length 3.
- $\overline{AM}$ is the reflection of $\overline{AM}$.
- $\overline{CB}$ is perpendicular to $\overleftrightarrow{AM}$, so $\angle AMC$ and $\angle AMB$ are right angles.
- $\angle BAM \cong \angle CAM$, so each angle measures 37°.
- $\angle C \cong \angle B$, and each angle measures $180° - (90° + 37°) = 53°$ (by the fact that the sum of the angles of a triangle is 180°).

In the Grade 7 Unit *Shapes and Designs*, students explored the angles made by a transversal cutting a pair of parallel lines. For some of the reasoning in this Unit, students will probably need to use ideas of vertical angles, supplemental angles, and alternate interior angles from *Shapes and Designs*. Those results are revisited and proven in this Unit as well.

In the diagram below, lines L_1 and L_2 are parallel lines cut by transversal L_3, and one angle measures 120°. From this, you can deduce all the other angle measures.



- $\angle d = 60°$ because it is supplementary to the 120° angle.
- $\angle a = 120°$ because it is supplementary to $\angle d$ (OR $\angle a = 120°$ because vertical angles are equal).

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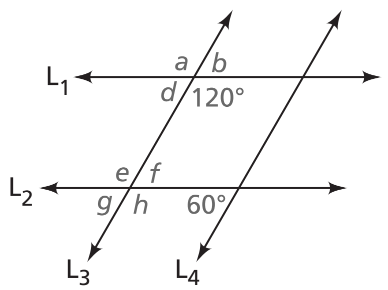
- $\angle b = 60°$ because it is supplementary to the 120° angle (OR $\angle b = \angle d = 60°$ because vertical angles are equal).
- $\angle h = 120°$ because you can translate the angle marked 120° along the transversal L_3 and match $\angle h$.
- $\angle f = 60°$ because it is supplementary to $\angle h$ (OR $\angle f = \angle d = 60°$ because alternate interior angles are equal).
- $\angle e = 120°$ because it is supplementary to $\angle f$ (OR $\angle \ = 120°$ because alternate interior angles are equal OR $\angle e = \angle h = 120°$ because vertical angles are equal).
- $\angle g = 60°$ because it is supplementary to $\angle h$ (OR $\angle g = \angle f = 60°$ because vertical angles are equal).

Suppose another angle measure and line L_4 are added to the diagram. We are not given that line L_4 is parallel to line L_3.

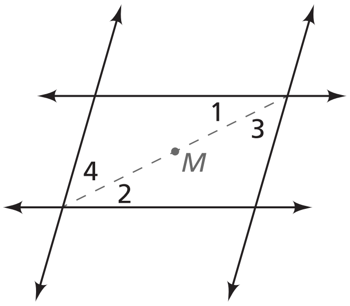


If $\angle g = 60°$ is translated along the transversal L_2, it will exactly match the angle marked 60°. So L_3, which is a side of $\angle g$, must be parallel to line L_4.

That is, you can use the ideas of transformations and the relationships among the angles formed when a transversal intersects two lines to determine whether or not the lines are parallel.

The relationships among parallel lines and their respective transversals can help especially when reasoning about parallelograms. For example, in the parallelogram shown below, you know, by definition, that there are two pairs of parallel lines and transversals.

Adding a diagonal to the parallelogram gives a third transversal and more congruent angles. That is, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ because alternate interior angles are equal.



You can rotate the parallelogram about the midpoint *M* of the diagonal to show that the diagonal divides the parallelogram into two congruent triangles. **Note:** ASA gives the same conclusion.

This reasoning from parallel lines cut by a transversal, combined with congruence and symmetry, leads to further results about parallelograms:

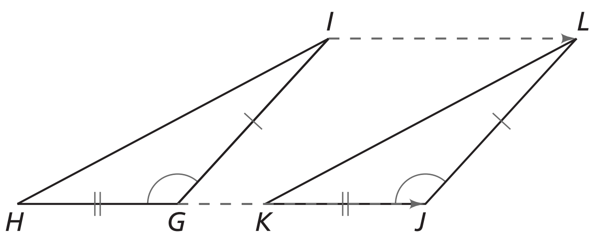
A. If *ABCD* is a parallelogram, then opposite sides are congruent.

B. If opposite sides of a quadrilateral are congruent, then it must be a parallelogram. **Note:** This is the converse of Statement A. The converse reverses the two parts of a logical statement.

C. The diagonals of a parallelogram bisect each other.

D. If the diagonals of a quadrilateral bisect each other, then that figure must be a parallelogram. **Note:** This is the converse of Statement C.

Similar reasoning with translations helps to prove that matching two pairs of corresponding sides and the pair of included angles in two triangles is enough to show that the triangles are congruent.



In the diagram above, if you translate triangle *HGI* so that $\angle G \rightarrow \angle J$, then you also have the following:

- $\overline{GI} \rightarrow \overline{JL}$ because $\overline{GI} \parallel \overline{JL}$ and $\overline{GI} \cong \overline{JL}$
- $\overline{HG} \rightarrow \overline{KJ}$ because $\overline{HG} \parallel \overline{KJ}$ and $\overline{HG} \cong \overline{KJ}$

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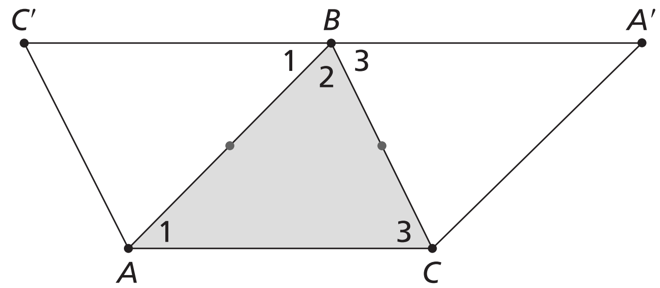
As with all translations of a segment, a parallelogram *GJLI* will be formed. To complete the rest of the congruence statements and to show that the triangles are congruent, check the following:

- $\overline{HI} \rightarrow \overline{KL}$
- $\angle H \rightarrow \angle K$
- $\angle I \rightarrow \angle L$

Since $\angle H \rightarrow \angle K$ and $\angle I \rightarrow \angle L$ under this translation, $\overline{HI} \rightarrow \overline{KL}$ and $\overline{HI} \parallel \overline{KL}$. Therefore, $\angle H \cong \angle K$ because they are corresponding angles of parallel lines cut by a transversal. Finally, since the sum of the measures of the angles of a triangle is 180°, $\angle I \cong \angle L$. To conclude, given that two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, *all* corresponding parts of the triangles must be congruent.

In addition to the above applications, you can also use ideas from transformational geometry to prove that the sum of the angles of a triangle is 180°.

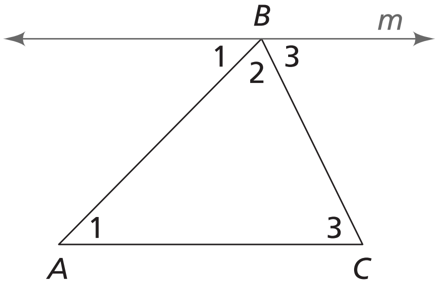
Start with triangle *ABC* and angles 1, 2, and 3. Rotate the triangle 180° about the midpoints of $\overline{AB}$ and $\overline{BC}$.



Then $\overline{A'B}$ is parallel to $\overline{AC}$, and $\overline{BC'}$ is parallel to $\overline{AC}$. Points A', B, and C' must be collinear since $\overline{A'B}$ and $\overline{BC'}$ are both parallel to $\overline{AC}$ and share a common point *B*. So, $\overline{A'C'}$ is a straight line through point *B*. Thus, $\angle 1 + \angle 2 + \angle 3 = 180°$.

You can also use Euclidean geometry to prove that the sum of the angles of a triangle is 180°.

Start with triangle ABC. Add a line m through point B that is parallel to $\overline{AC}$.



The angles marked "3" are congruent because alternate interior angles are equal. The angles marked "1" are congruent for the same reason. Thus, $\angle 1 + \angle 2 + \angle 3 = 180°$ because they form a straight angle.

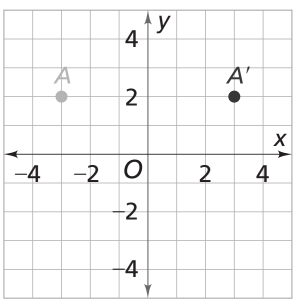
Coordinate Rules for Symmetry Transformations

In Investigation 3, we return to transformations and look at transformations of figures on a coordinate plane.

Reflections

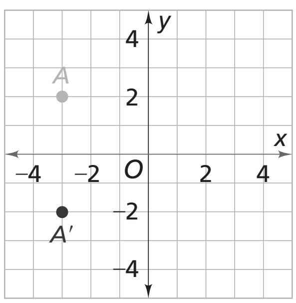
Students write rules for describing reflections of figures drawn on a coordinate grid. The rules tell how to find the image of a general point (x, y) under a reflection. For example, a reflection in the y-axis matches (x, y) to $(-x, y)$; a reflection in the x-axis matches (x, y) to $(x, -y)$; and a reflection in the line $y = x$ matches (x, y) to (y, x).

Reflection in the *y*-axis



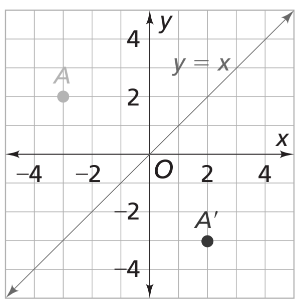
$(x, y) \rightarrow (-x, y)$

Reflection in the *x*-axis



$(x, y) \rightarrow (x, -y)$

Reflection in the Line *y* = *x*



$(x, y) \rightarrow (y, x)$

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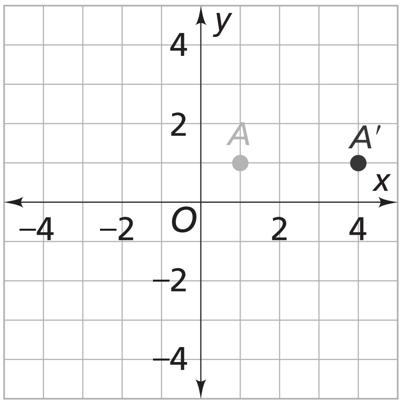
Translations

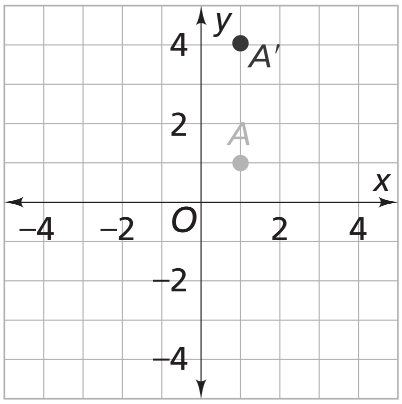
A translation can also be specified by a rule for locating the image of a general point (x, y). For example, a horizontal translation of 3 units to the right matches (x, y) to ($x + 3$, y), and a vertical translation of 3 units up matches point (x, y) to (x, $y + 3$). A translation along an oblique line can be specified with the vertical and horizontal components of the slide. For example, a translation in the direction of the line $y = x$, 2 units right and 2 units up, matches (x, y) to ($x + 2$, $y + 2$). A translation of 2 units to the right and 4 units down matches (x, y) to ($x + 2$, $y - 4$).

3 Units Right



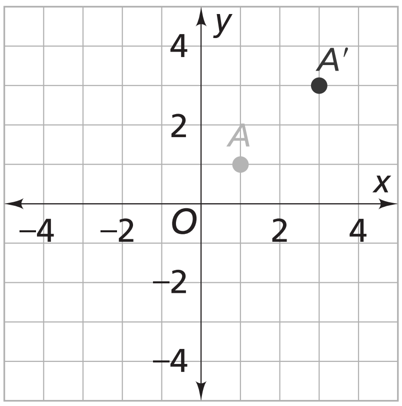
$(x, y) \rightarrow (x + 3, y)$

3 Units Up



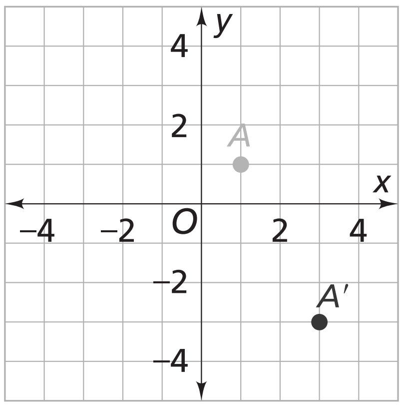
$(x, y) \rightarrow (x, y + 3)$

2 Units Right, 2 Units Up



$(x, y) \rightarrow (x + 2, y + 2)$

2 Units Right, 4 Units Down

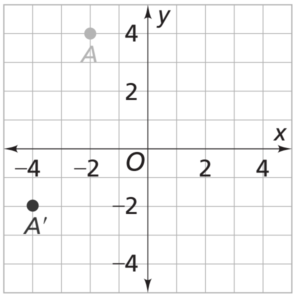


$(x, y) \rightarrow (x + 2, y - 4)$

Rotations

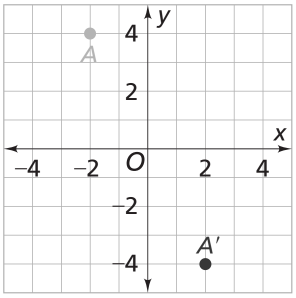
As with reflections and translations, students learn to specify certain rotations by giving rules for locating the image of a general point (x, y). For example, a rotation of 90° about the origin matches the point (x, y) to the image point ($-y$, x), and a rotation of 180° about the origin matches (x, y) to ($-x$, $-y$).

90° Rotation About (0, 0)



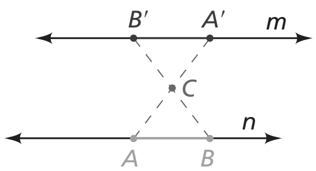
$(x, y) \rightarrow (-y, x)$

180° Rotation About (0, 0)



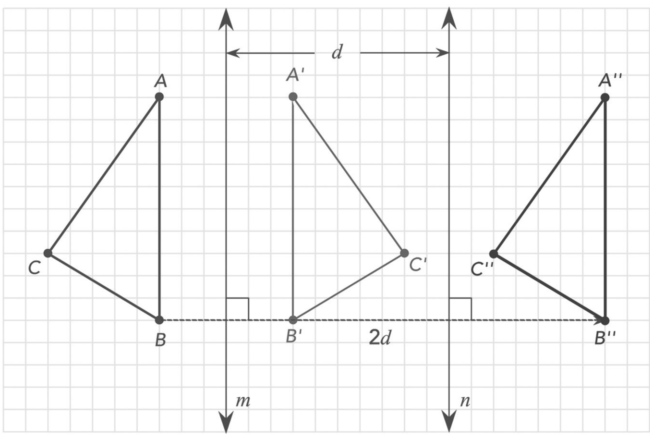
$(x, y) \rightarrow (-x, -y)$

As a consequence of using coordinate rules to define the results of transformations on points, students can verify observations they made previously about distances and slopes. For example, a 180° rotation of $\overline{AB}$ about a point C will result in an image $\overline{A'B'}$ that is parallel and congruent to $\overline{AB}$. This observation becomes very useful for proving further results.



Combining Transformations

In very informal ways, students explore combinations of transformations. In a few instances in the ACE Extensions, students are asked to describe a single transformation that will give the same result as a given combination. For example, reflecting a figure in a line and then reflecting the image in a parallel line has the same result as translating the figure in a direction perpendicular to the reflection lines for a distance equal to twice the distance between the lines. Visit Teacher Place at mathdashboard.com/cmp3 to see the complete video.



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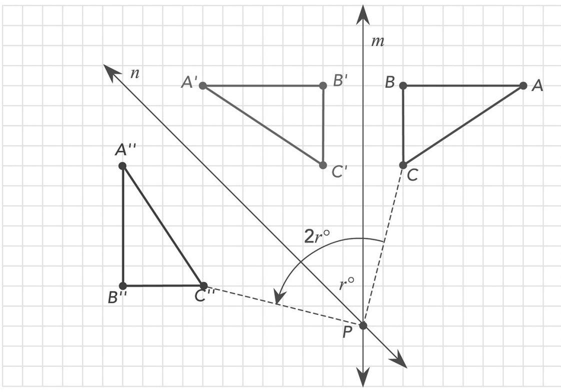
Look for these icons that point to enhanced content in *Teacher Place*   Video   Interactive Content

Reflecting a figure in a line and then reflecting the image in an intersecting line has the same result as rotating the original figure about the intersection point of the lines by an angle equal to twice the angle formed by the reflection lines. Notice that reflecting the triangle *ABC* in line 1 and then reflecting the image *A′B′C′* in line 2 does NOT give the same result as reflecting triangle *ABC* in line 2 first and then reflecting the image in line 1. Visit Teacher Place at mathdashboard.com/cmp3 to see the complete video.



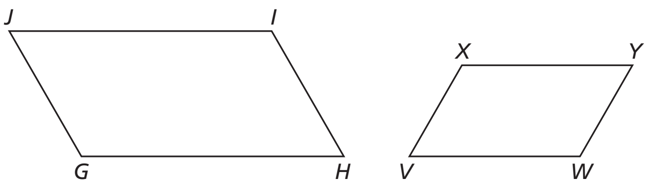
Similarity

In everyday language the word *similar* is used to suggest that objects or ideas are alike in some way. In mathematical geometry, the word *similar* is used to describe figures that have the same shape but different size. You can formally define the term with the concepts and language of transformations.

Two figures are similar if

- the measures of their corresponding angles are equal
- there is a constant factor, called the scale factor, with which the lengths of the sides of one figure can be multiplied by to give the lengths of the corresponding sides in the other figure

Parallelograms *GHIJ* and *VWXY* are similar.

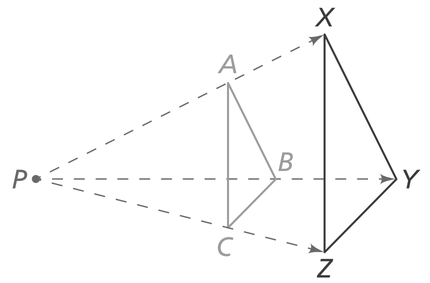


The corresponding angle measures of *GHIJ* and *VWXY* are equal. The side lengths in *VWXY*, when multiplied by 1.5, equal the corresponding side lengths in *GHIJ*. Thus, the scale factor from *VWXY* to *GHIJ* is $\frac{3}{2}$, or 1.5. *VWXY* stretches, or is enlarged, to become *GHIJ*. You can also say that the scale factor from *GHIJ* to *VWXY* is $\frac{1}{1.5}$, or $\frac{2}{3}$. *GHIJ* shrinks, or is reduced, to become *VWXY*.

Note: Congruent figures are a special case of similar figures with a scale factor of 1.

Dilations

A *dilation* with center P and scale factor $k > 0$ is a transformation of the plane (or space) that maps each point X to a point X' on ray PX so that $PX' = kPX$. The center point of any dilation maps to itself. The following diagram shows a dilation centered at P with scale factor $\frac{3}{2}$ that maps point A to point X, point B to point Y, and point C to point Z. The same diagram also shows how a dilation with scale factor $\frac{2}{3}$ maps point X to point A, point Y to point B, and point Z to point C.



In everyday language, the word *dilation* usually suggests enlargement. However, in standard mathematical usage, the word dilation is used to describe either an enlargement or stretching action (scale factor greater than 1) or a reduction or shrinking action (scale factor between 0 and 1).

There are several very important properties of all dilation transformations. For any dilation with center P and scale factor k:

- If the dilation maps point X to point X' and point Y to point Y', then it maps segment XY onto segment $X'Y'$ and $X'Y' = kXY$.
- If the dilation maps $\angle ABC$ onto $\angle A'B'C'$, then the two angles are congruent.
- If the dilation maps polygon or circle F onto polygon or circle F', then the perimeter of F' is k times the perimeter of F and the area of F' is k^2 times the area of F.

Taken together, these properties explain how dilations preserve shapes of figures by preserving angle measures and proportional side length relationships among corresponding parts.

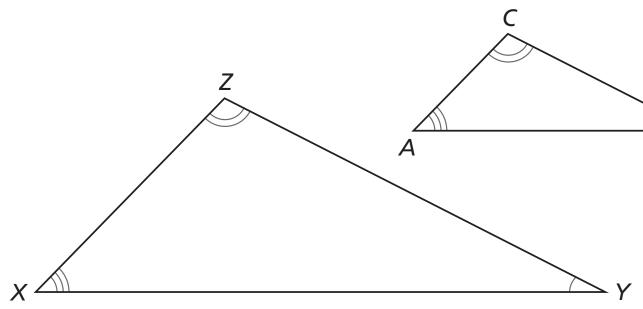
Coordinate Rules for Dilations

The coordinate rule to transform a figure to a similar image is $(x, y) \rightarrow (kx, ky)$ for some scale factor k. For example, if a figure is dilated by a scale factor of 4 about the origin, the coordinate rule is $(x, y) \rightarrow (4x, 4y)$. In general, if a figure is dilated by a scale factor of k about center (a, b), the coordinate rules are of the form $(x, y) \rightarrow (kx + a, ky + b)$. These coordinate rules are called *similarity transformations*.

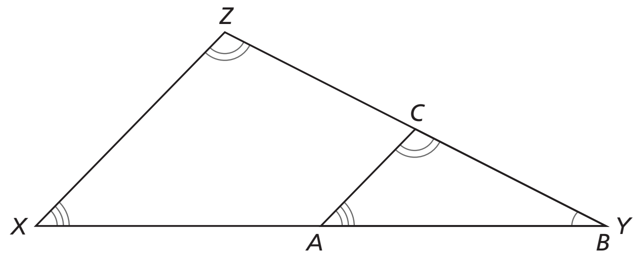
Similar Figures

In geometry, you encounter two figures like triangles, rectangles, or trapezoids that seem to have the same shape but different size. Oftentimes, they are positioned so that you cannot see a simple dilation that would map one figure one onto the other. You first need to flip, turn, and/or slide the smaller figure onto the larger figure. For example, in triangles *ABC* and *XYZ* shown below, you can measure to check that:

$\angle X \cong \angle A$	$XY = 2 \cdot AB$
$\angle Y \cong \angle B$	$YZ = 2 \cdot BC$
$\angle Z \cong \angle C$	$ZX = 2 \cdot CA$



However, no simple dilation will map one of the triangles onto the other. You can use a sequence of translations to move the smaller triangle onto the larger triangle.



Then you can use a dilation centered at *Y* to transform triangle *ABC* exactly onto triangle *XYZ*, confirming that the original triangles are similar.

This illustrates the general definition of similarity for geometric figures: *Two figures are similar if there is a combination of one or more rigid motions and a dilation that transform one figure exactly onto the other.*

In the special case of triangles, it is not necessary to construct the sequence of rigid motions and dilation to confirm similarity. It turns out that critical knowledge about the measures of angles and sides of two triangles is sufficient to guarantee similarity. The most common conditions are:

- If two angles of one triangle are congruent to two corresponding angles of another, then the triangles are similar. (Angle-Angle)

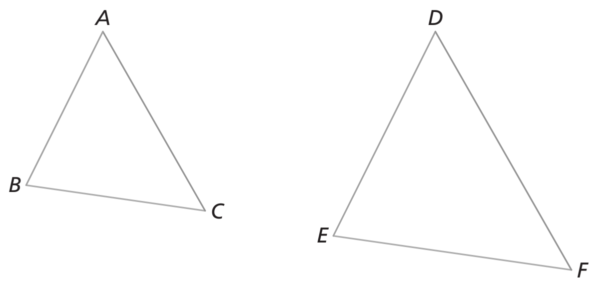
Note: The congruence of two pairs of angles guarantees congruence of the third pair because the sum of angle measures in any triangle is 180°.

- If the ratios of the lengths of three pairs of corresponding sides in two triangles are equal, then the triangles are similar. (Side-Side-Side)

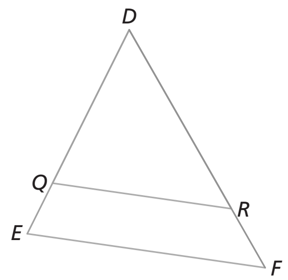
The SSS Similarity Theorem

The Side-Side-Side Similarity Theorem states that, if corresponding sides of two triangles are proportional, then the triangles are similar.

Take triangles *ABC* and *DEF* to have corresponding sides that are proportional, such that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.



You can draw a line *QR* such that the line is equal in length to line *BC* and parallel to line *EF*.



Since *QR* and *EF* are parallel, angles *Q* and *E* are congruent, as are angles *R* and *F*. So, triangles *DQR* and *DEF* are similar.

Since triangles *DQR* and *DEF* are similar, then $\frac{DQ}{DE} = \frac{DR}{DF} = \frac{QR}{EF}$.

We know that *AB* must be congruent to *DQ*, and *AC* must be congruent to *DR*. These facts result from the two proportions $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ and $\frac{DQ}{DE} = \frac{DR}{DF} = \frac{QR}{EF}$, and the fact that $QR = BC$.

Since all corresponding line segments are congruent for triangles *ABC* and *DQR*, we can assume that the triangles are congruent by the Side-Side-Side Congruence Postulate. And, as similarity is transitive, since *DQR* is similar to *DEF*, its congruent triangle *ABC* must also be similar to *DEF*.

- If the ratios of lengths of two pairs of corresponding sides in two triangles are equal and the angles included in those sides are congruent, then the triangles are similar. (Side-Angle-Side)

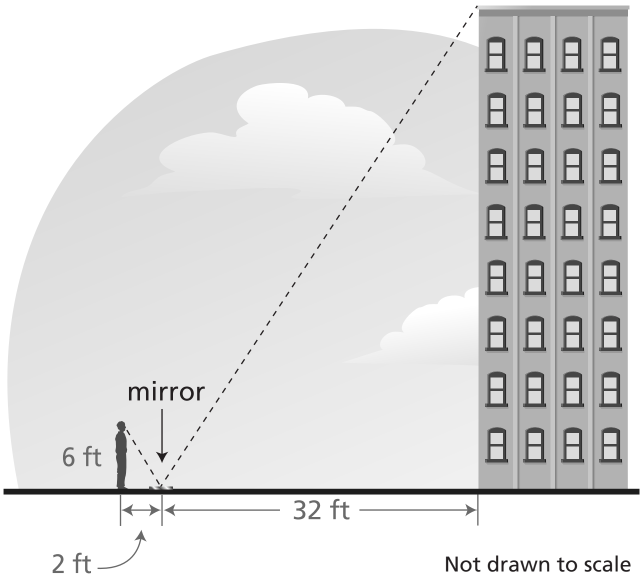
continued on next page

When the scale factor of the dilation in a similarity transformation is equal to 1, the figures being compared are congruent. So, congruence is really a special case of similarity.

Applications of Similarity

You can use the relationships between corresponding parts of similar triangles to deduce unknown side lengths of one of the triangles. This application of similarity is especially useful in situations where you cannot measure a length or height directly.

For example, the diagram below shows a method for calculating the height of an object that you cannot reach with a ruler or tape measure. The observer places a mirror on the ground and stands a certain distance from the mirror where he/she sees the top of the building in the mirror. The line from the observer's eye meets the ground at the same angle as the line from the top of the building to the mirror.



The two triangles outlined in the diagram are similar. You can measure the observer's height, his/her distance from the mirror, and the distance from the mirror to the base of the building, as shown. You can then use these measurements and proportional reasoning to find the height of the building x. That is, $\frac{x}{6} = \frac{32}{2}$. So, the height of the building is 92 feet.