UNDERSTANDING THE NATURE OF UNCERTAINTY IN PROBLEM SOLVING SITUATIONS

Taren Going
Michigan State University
goingtar@msu.edu

Merve Kursav
Michigan State University
kursavme@msu.edu

Yvonne Slanger-Grant
Michigan State University
granty@msu.edu

Kristen Bieda
Michigan State University
kbieda@msu.edu

Alden J. Edson
Michigan State University
edsona@msu.edu

Productive disciplinary engagement (PDE; Engle & Conant, 2002) describes classroom situations where students publicly engage in disciplinary practices. Researchers have argued that PDE is fostered when students engage in problematizing, where they grapple with genuine uncertainty about mathematical objects, among other characteristics. Building on work by Zaslavsky, this paper advances a framework to capture the nature of uncertainty in mathematics classrooms.

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Introduction

Problematizing is a core principle of classrooms that embody *productive disciplinary engagement* (PDE) (Engle & Conant, 2002). Engle (2011) articulates problematizing as situations involving "uncertainties...related to the discipline...to be taken up by students." Although some existing work has operationalized other principles of PDE, such as authority and accountability, measuring the extent to which students are engaging in problematizing is less well-defined in existing literature. While existing research has modeled uncertainty when students participate in group work (Wood & Kalinec, 2012) and the linguistic basis for verbal expressions of uncertainty (Martin & Rose, 2003; Rowland, 1995), uncertainties that arise during problematizing are tied to the mathematical work students are engaged in. Engle (2011) suggests that uncertainties which arise in situations of PDE include (1) competing claims; (2) unknown path; (3) questionable conclusion; (4) non-readily verifiable outcome; and (5) other. Our work seeks to extend these initial categories to generate a comprehensive framework of situations of uncertainty when students grapple with open mathematical problems.

Methods

To generate the framework, we used an iterative process to code video recordings of classroom episodes where students were engaged in solving open problems. Our video data came from one lesson taught across two class periods in an 8th grade mathematics classroom in a rural school setting, collected in the late spring. The lesson was based on an investigation in the Connected Mathematics (CMP) curriculum (Lappan, Fey, Fitzgerald, Friel & Phillips, 2006), a curriculum used in this school from 6^{th} to 8^{th} grades. The teacher had taught from CMP in prior years. The specific lesson features students engaged in finding and generalizing patterns to find an algebraic expression for the number of tiles around a square pool, given any side length. The video shows the teacher's launch of the problem, small group work as students explore the problem, and a summary discussion of different groups' solutions and strategies.

We used iterative processes (Shaffer, 2017) to generate categories in our framework. The process began with individual open coding by each member of the research team to describe the uncertainties observed through students' spoken words, gestures, postures, and silences. Next, we engaged in several iterative cycles of axial coding (Strauss & Corbin, 1998) to collectively compare codes against the data and existing codes in order to restructure and refine the framework to account for students' processes. In the final iterative cycles, we looked for overlap in our descriptions of types of uncertainty and worked to eliminate any redundancies. We then compiled the framework and individually re-coded the video data to determine that all uncertainties could be coded with the refined framework.

The Framework

The process described in the Methods section yielded the framework shown in Figure 1. Building from work by Engle (2011) and Zaslavsky (2005), our modified categories include uncertainty about: (1) What action to take - students have uncertainty about what to do, where they may feel unable to begin or continue working; (2) Justifying actions or outcomes - students don't know how to verify, check, or justify what they are doing or some outcome of mathematical work; (3) Meaning of conclusion - students have or are presented with a final result, but are uncertain about what it means or how to apply it more broadly; and (4) Competing alternatives - two or more mathematical ideas arise that cause hesitation or uncertainty. Competing alternatives may occur during any stage of problem solving and is not necessarily independent of other types of uncertainty. Further, uncertainties do not necessarily arise in a linear fashion, and the resolution of one uncertainty might give rise to others.

Categories of Uncertainty	
Code	**Definition**
A. What Action to Take	
1. What a mathematical object represents	Students have hesitation/questioning about the possible meaning conveyed by a mathematical object (i.e. a diagram, shape, expression/equation, graph, table)
2. What does [part of problem context] mean?	Students are unsure about what is meant by some detail of the problem statement or part of the problem context.
3. How to create or use something	Students are uncertain about how to use the tools they have to achieve their desired goal.
4. What is the goal? What is to be created?	Students are unsure about what the problem is asking them to do or what they are trying to accomplish.
B. Justifying Actions or Outcomes	
5. Connecting to prior work	Students are uncertain on which prior knowledge they can draw.
6. Whether strategy is productive	Students are pursuing one method or strategy but are unsure if it will help them achieve their desired mathematical goal or solve the problem.
7. Representing thinking	Students have a partially developed idea or conjecture but are not sure how to represent it physically or verbally.
8. Justifying strategy	Students are uncertain of how to justify that the mathematical actions they have taken are correct.
9. Has solution been reached?	Students are unsure if the solution has been reached or if there is more to do.

C. Meaning of Conclusion	
10. What to conclude	Students are uncertain of their solution's broader meaning or why it works.
11. Whether solution makes sense	Students are unsure if their solution makes sense or is reasonable, or how to determine this or explain it to someone else.
12. Strategy works for other cases	Students have an idea or conjecture, but are unsure what it means more generally or in other contexts
D. Competing Alternatives	
13. Relating one mathematical object to another	Students are uncertain if two mathematical objects are related or how they might be related.
14. Finding another way to represent idea/work	Students are uncertain if there is another way to represent their existing mathematical idea/work, or how they would go about doing that.
15. Revising work	Students are uncertain of how to revise or change their work or strategy, once they recognize it needs to be revised.
16. Finding another way to solve problem	Students are uncertain if there is another way to solve the problem, or how they would go about doing that.
17. Seeing commonality among strategies	Students are uncertain if two mathematical strategies are related or how they might be related.

Figure 1. The framework and definitions

Using the Framework: Capturing Uncertainty when Problematizing the Pool Problem

We present some of the uncertainties that arose in one pair of students' efforts on the Pool Problem (Lappan et al., 2006a, p. 6), where they are asked to find an algebraic expression for the number of tiles around a square pool given any side length. This focal pair of students initially created an algebraic expression to determine the correct answer for a 2x2 pool, but when asked if it would work for a larger pool, one student said "I'm thinking… but probably not." They struggled to calculate appropriate examples with the expression s^2+4s+s, unsure which sizes of the pool it might work for. This illustrates the uncertainty Whether a Strategy is Productive (B-6). With the teacher's assistance, the pair tried their expression with a larger pool and determined it was incorrect. One of the students attempted to explain where the structure of their expression came from, saying "I don't know, I tried to take the… what's it called… not the expanded form…", and the teacher asks, "The factored form?" The student agreed, but that they were not sure if it would apply. This illustrates the uncertainty Connecting to Prior Work (B-5). The teacher then directed the pair's attention to the initial problem statement and the use of the variable s, and one student quickly gestured to a diagram in the textbook and asked, "Is it the outside side or just the inside one?" This uncertainty about the labeling of a diagram illustrates What a Mathematical Object Represents (A-1). Moreover, this example shows how addressing one uncertainty may involve one or more additional uncertainties arising.

After resolving this uncertainty about the initial diagram, the teacher asked the pair about an equation they wrote down earlier. One of the students confirmed that they used 4s to represent the sides, and the teacher asked, "Now what do you need?" The student responded, "The… corners." The teacher asked, "So how could you write that into the equation?" and the student answered, "Um… maybe you could, like, put it as a second power?" The student had a mental representation of the problem relating to sides and corners but struggled to represent it as an equation, illustrating the uncertainty Representing Thinking (B-7). The pair knew that they needed to include 4s in their expression, but they were not sure how to modify this expression to account for the corners. One of the students began to try various other multipliers in their calculator and realized that multiplying by a different number for each pool size produced the correct number of tiles. However, the pair tentatively proceeded to produce what they considered as an appropriate equation. This illustrates the uncertainty Revising Work (D-15).

In many cases, not all uncertainties were resolved quickly. Uncertainty about revising work, for example, extended through the rest of the small group work time. To further understand the nature of problematizing, we acknowledge several key categories of meta-data important to correlate with codes for uncertainty, such as *source* (others, technology teacher, self), *indicator* (gesture, statement, question), *phase* (launch, explore, summary), *part of problem* (initial challenge, explore, check, verify), and *number of students* (whole group, small group, individual).

Discussion

An essential part of achieving the goal of increasing students' opportunities to engage in PDE is understanding the mechanisms that allow PDE to emerge in classroom settings. In this paper, we advance a framework that may help the field get closer to identifying situations of problematizing that are essential for PDE by articulating kinds of uncertainty that are empirically grounded from a video-recorded sample of classrooms engaging in rich, open problem solving. This framework provides a starting point for further research, including investigating problematizing as it arises in classrooms with enhanced technology supports for collaborative problem solving as well investigating the relationship between authority and accountability and the emergence or scarcity of particular kinds of uncertainty.

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