Focus Questions

Background

The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

Description

The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

Purpose

These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.

7-6 What Do You Expect

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Experimental and Theoretical Probabilities Understand experimental and theoretical probabilities

- Recognize that probabilities are useful for predicting what will happen over the long run
- For an event described in everyday language, identify the outcomes in a sample space that compose the event
- Interpret experimental and theoretical probabilities and the relationship between them and recognize that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers
- Distinguish between outcomes that are equally likely or not equally likely by collecting data and analyzing experimental probabilities
- Realize that the probability of simple events is a ratio of favorable outcomes to all outcomes in the sample space
- Recognize that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
- Determine the fairness of a game

Reasoning With Probability Explore and develop probability models by identifying possible outcomes and analyze probabilities to solve problems

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process
- Represent sample spaces for simple and compound events and find probabilities using organized lists, tables, tree diagrams, area models, and simulation
- Realize that, just as with simple events, the probability of a compound event is a ratio of favorable outcomes to all outcomes in the sample space
- Design and use a simulation to generate frequencies for simple and compound events
- Analyze situations that involve two or more stages (or actions) called *compound events*
- Use area models to analyze the theoretical probabilities for two-stage outcomes
- Analyze situations that involve binomial outcomes

- Use probability to calculate the long-term average of a game of chance
- Determine the expected value of a probability situation
- Use probability and expected value to make a decision

Focus Questions and Mathematical Reflections

Investigation 1 A First Look at Chance	**Investigation 2** Experimental and Theoretical Probability	**Investigation 3** Making Decisions With Probability	**Investigation 4** Analyzing Compound Events Using an Area Model	**Investigation 5** Binomial Outcomes
Problem 1.1 Choosing Cereal: Tossing a Coin to Find Probabilities **Focus Question** How does collecting more data help you predict the outcome of a situation? **Problem 1.2** Tossing Paper Cups: Finding More Probabilities **Focus Question** How does modeling with an experiment help you determine possible outcomes and the likelihood of each outcome? **Problem 1.3** One More Try: Finding Experimental Probabilities **Focus Question** How do you determine the relative frequency of an outcome? **Problem 1.4** Analyzing Events: Understanding Equally Likely **Focus Question** How can you determine whether the outcomes of a probability event are all equally likely, and why would this information matter?	**Problem 2.1** Predicting to Win: Finding Theoretical Probabilities **Focus Question** How does experimental probability compare to theoretical probability for a given situation? **Problem 2.2** Choosing Marbles: Developing Probability Models **Focus Question** What are some properties of theoretical probabilities? **Problem 2.3** Designing a Fair Game: Pondering Possible and Probable **Focus Question** How can you decide whether a game is fair or not? **Problem 2.4** Winning the Bonus Prize: Using Strategies to Find Theoretical Probabilities **Focus Question** How can you determine all of the probabilities for a compound event?	**Problem 3.1** Designing a Spinner to Find Probabilities **Focus Question** How do you determine probability using a spinner? **Problem 3.2** Making Decisions: Analyzing Fairness **Focus Question** When using a tool to simulate a fair game, what things must you consider? **Problem 3.3** Roller Derby: Analyzing a Game **Focus Question** How does understanding probability help you design a winning strategy? **Problem 3.4** Scratching Spots: Designing and Using a Simulation **Focus Question** How can you design a simulation to determine probability?	**Problem 4.1** Drawing Area Models to Find the Sample Space **Focus Question** How can an area model represent a situation to help analyze probabilities? **Problem 4.2** Making Purple: Area Models and Probability **Focus Question** How can you use experimental or theoretical probabilities of a compound event to predict the number of times one particular combination will occur out of any given number of repetitions of the event? **Problem 4.3** One-and-One Free Throws: Simulating a Probability Situation **Focus Question** How is an area model for the one-and-one free-throw situation like or unlike the area model for the Making Purple game? **Problem 4.4** Finding Expected Value **Focus Question** How is expected value different from probabilities of outcomes?	**Problem 5.1** Guessing Answers: Finding More Expected Values **Focus Question** If you do not know the answers to a true/false test, what is the probability that you can get a good score with random guesses? **Problem 5.2** Ortonville Binomial Probability **Focus Question** What patterns are there in models for binomial probability situations that are equally likely? How do these patterns help you answer probability questions? **Problem 5.3** A Baseball Series: Expanding Binomial Probability **Focus Question** If two teams are evenly matched, how do binomial probabilities help you figure out the probabilities that a winner of the required number of games will occur after a certain number of games?
Mathematical Reflection 1. How do you find the experimental probability that a particular result will occur? Why is it called the experimental probability? 2. In an experiment, are 30 trials as good as 500 trials to predict the chances of a result? Explain/ 3. What does it mean for results to be equally likely?	**Mathematical Reflection** 1. Describe how you can find the theoretical probability of an outcome. Why is it called theoretical probability? 2. a. Suppose two people do an experiment to estimate the probability of an outcome. Will they get the same probabilities? Explain. b. Two people analyze a situation to find the theoretical probability of an outcome. Will they get the same probabilities? Explain. c. One person uses an experiment to estimate	**Mathematical Reflection** 1. Describe a situation in which you and a friend can use probability to make a decision. Can the probabilities of the outcomes be determined both experimentally and theoretically? Why or why not? 2. Describe a situation in which it is difficult or impossible to find the theoretical probabilities of the outcomes. 3. Explain what it means of a probability situation to be fair.	**Mathematical Reflection** 1. Describe four probability situations that involve two actions. Describe the outcomes for these situations. 2. You can use an area model or a simulation to determine the probability of a situation that involves two actions. Explain how each of these is used. 3. Describe how you would calculate the expected value for a probability situation. 4. Expected value is sometimes called	**Mathematical Reflection** 1. Describe five different binomial situations. Explain why they are binomial situations. 2. Tossing a coin three times is an example of a situation involving a series of three actions, each with two equally likely outcomes. a. Pick one of the situation in Question 1. Describe a series of three actions, each with two equally likely outcomes. Make a list of all the possible outcomes. b. Write a question about your situation that can be answered by your list.

	the probability of an outcome. Another person analyzes the situation to find the theoretical probability of the outcome. Will they get the same probabilities? Explain. 3. What does it mean for a game to be fair? 4. What is a sample space, and how can it be represented?	4. Describe some of the strategies for determining the theoretical probabilities for situations in this unit. Give an example of a situation for each of the strategies.	the longer-term average. Explain why this makes sense.	3. As you increase the number of actions for a binomial situation, what happens to the total number of possible outcomes? For example, suppose you increase the number of times a coin is tossed. What happens to the total number of outcomes?