Focus Questions

Background

The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

Description

The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

Purpose

These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.

7-7: Filling and Wrapping

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Surface Areas and Volumes of Polygonal Prisms and Cylinders Understand surface areas and volumes of prisms and cylinders and how they are related

- Describe prisms by using their vertices, faces, and edges
- Visualize three-dimensional shapes and the effects of slicing those shapes by planes
- Deepen understanding of volumes and surface areas of rectangular prisms
- Estimate and calculate surface areas and volumes of polygonal prisms by relating them to rectangular prisms
- Explore the relationships between the surface areas and volumes of prisms
- Relate surface areas and volumes for common figures, especially optimization of surface area for fixed volume
- Predict the effects of scaling dimensions on linear, surface area, and volume measures of prisms, cylinders, and other figures
- Investigate the relationship between volumes of prisms and volumes of cylinders as well as the relationship between surface areas of prisms and surface areas of cylinders
- Use volumes and surface areas of prisms to develop formulas for volumes and surface areas of cylinders
- Discover that volumes of prisms and cylinders can be calculated as the product of the area of the base and the height
- Solve problems involving surface areas and volumes of solid figures

Areas and Circumferences of Circles Understand the areas and circumferences of circles and how they are related

- Relate area of a circle to covering a figure and circumference to surrounding a figure
- Estimate and calculate areas and circumferences of circles
- Explore the relationship between circle radius (or diameter) and circumference
- Explore the relationship between circle radius (or diameter) and area
- Investigate the connection of π to area calculation by estimating the number of radius squares needed to cover a circle
- Investigate the relationship between area and circumference of a circle
- Solve problems involving areas and circumferences of circles

Volumes of Spheres and Cones Understand the relationships between the volumes of cylinders and the volumes of cones and spheres

- Relate volumes of cylinders to volumes of cones and spheres
- Estimate and calculate volumes of spheres and cones
- Solve problems involving surface areas and volumes of spheres and cones

Focus Questions and Mathematical Reflections

Investigation 1 **Building Smart Boxes: Rectangular Prisms**	**Investigation 2** **Polygonal Prisms**	**Investigation 3** **Area and Circumference of Circles**	**Investigation 4** **Cylinders, Cones, and Spheres**
Problem 1.1 How Big Are Those Boxes? Finding Volume **Focus Question:** How do you calculate the surface area and volume of a rectangular prism?	**Problem 2.1** Folding Paper: Surface Area and Volume of Prisms **Focus Question:** For a prism with fixed height and fixed lateral area, how do the volume and surface area of the prism change as the number of sides increases?	**Problem 3.1** Going Around in Circles: Circumference **Focus Question:** What is the relationship between the diameter or radius of a circle and its circumference?	**Problem 4.1** Networking: Surface Area of Cylinders **Focus Question:** How can you calculate the surface area of a cylinder? Why does that strategy work?
Problem 1.2 Optimal Containers I: Finding Surface Area **Focus Question:** Suppose you design a box in the shape of a rectangular prism with a volume of 24 cm^3. What are the shape and dimensions of the box that has minimum surface area?	**Problem 2.2** Packing a Prism: Calculating Volume of Prisms **Focus Question:** What general strategy can be used to find the volume of any prism—triangular, rectangular, pentagonal, and so on?	**Problem 3.2** Pricing Pizza: Connecting Area, Diameter, and Radius **Focus Question:** How does the area of a circle increase as the circle's radius and diameter increase?	**Problem 4.2** Wrapping Paper: Volume of Cylinders **Focus Question:** How can you calculate the volume of a cylinder? How is the procedure similar to calculating the volume of a prism?
Problem 1.3 Optimal Containers II: Finding the Least Surface Area **Focus Question:** What are the dimensions of the rectangular prism that has the least surface	**Problem 2.3** Slicing Prisms and Pyramids **Focus Question:** What surface shapes and three-dimensional figures can be created by slicing a rectangular prism in various	**Problem 3.3** Squaring a Circle to Find is Area **Focus Question:** What is the relationship between the area of a circle and its radius?	**Problem 4.3** Comparing Juice Containers: Comparing Surface Areas **Focus Question:** How does the surface area of a cylinder compare to the surface area of a

area for a given volume?	directions?		rectangular prism for a given volume?
Problem 1.4 Compost Containers: Scaling Up Prisms **Focus Question:** As you change the dimensions of a rectangular prism by a certain scale factor, how do the surface area and volume of the prism change?		**Problem 3.4** Connecting Circumference and Area **Focus Question:** What is the relationship between the circumference and area of a circle?	**Problem 4.4** Filling Cones and Spheres **Focus Question:** If a sphere and a cone have the same dimensions as a cylinder, how do the volumes compare? What formulas for volume of a sphere and the volume of a cone can you write using these relationships?
			Problem 4.5 Comparing Volumes of Spheres, Cylinders, and Cones **Focus Question:** What are some relationships you can use involving a cone, a sphere, and a cylinder with the same dimensions?
Mathematical Reflections: 1. How can you calculate the volume and surface area of a rectangular prism from measures of its length, width, and height? Explain why this works. 2. How are the surface area and volume of a rectangular prism related to each other? 3. How will the surface area and volume of a prism change in each of the following cases? a. You increase or reduce one dimension by a scale factor of *f*.	**Mathematical Reflections:** 1. How can you find the surface area of any right prism? Explain why your method works. 2. How can you find the volume of any right prism? Explain why your method works 3. What two- and three-dimensional shapes result when a right rectangular prism is cut by a. a horizontal slice? b. a vertical slice? c. a slanted slice?	**Mathematical Reflections:** 1. How can you find the circumference and area of a circle from measures of its radius or diameter? 2. How is the challenge of finding circumferences and areas of circles similar to that of finding perimeters and areas of polygons such as triangles, rectangles, and other parallelograms? In what ways are those tasks different?	**Mathematical Reflections:** 1. a. Compare the task of finding the circumference of the base and the surface area of a cylinder to that of finding the perimeter of the base and the surface area a prism. b. Compare the task of finding the volume of cylinders to that of finding the volume of prisms. c. How can you find the circumference of the base, the surface area, and the volume of a cylinder from measures of its radius or diameter and its

b. You increase or reduce two dimensions by a scale factor of *f*. c. You increase or reduce all three dimensions by a scale factor of *f*.			height? Explain why your formulas make sense. d. How do the surface area and the volume of a cylinder change if both the radius and height are changed by a factor of *f*? 2. a. How is the task of finding the volumes of spheres and cones similar to that of finding the volumes of prisms and cylinders? In what ways are those tasks different? b. How can you find the volume of a sphere or a cone from measures of its dimensions?