### **Focus Questions**

# **Background**

The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

# Description

The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

### **Purpose**

These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.

# 8-1: Thinking with Mathematical Models

Unit Goals, Focus Questions, and Mathematical Reflections

#### **Unit Goals**

### Recognize and model linear and nonlinear relationships in bivariate data

- Represent data patterns using graphs, tables, word descriptions and algebraic expressions
- Use mathematical models to answer questions about linear relationships
- Investigate the nature of linear variation in contexts
- Write linear functions from verbal, numerical, or graphical information
- Analyze, approximate, and solve linear equations
- Model situations with inequalities expressed as "at most" and "at least" situations
- Investigate the nature of inverse variation in contexts
- Use mathematical models to answer questions about inverse variation relationships

Compare inverse variation relationships with linear relationships

# **Characterize the Strength and Variability of Mathematical Models**

- Use data patterns to make predictions
- Fit a line to data that show a linear trend and measure goodness of fit
- Analyze scatter plots of bivariate data to determine the strength of the linear relationship between the two variables.
- Use correlation coefficients informally to describe the strength of the linear relationship illustrated by scatter plots.
- Distinguish between categorical and numerical variables.
- Use 2-way tables and analysis of cell frequencies and relative frequencies to help in deciding whether two categorical variables are related.
- Use standard deviation to measure variability in data distributions

**Focus Questions and Mathematical Reflections** 

Exploring Data Patterns	Linear Models and Equations	Inverse Variation	Variability and Associations in Numerical Data	Variability and Associations in Categorical Data
1.1 Bridge Thickness and Strength  Focus Question: How would you describe the relationship between bridge strength and bridge thickness revealed by your experiment?	2.1 Modeling Linear Data Patterns  Focus Question: How can you find a linear function that is a good model for a set of data and then measure the accuracy of that model with residuals?	3.1 Rectangles with Fixed Area  Focus Question: When the product of two variables is some fixed number, what is the pattern of change and how is that pattern of change reflected in tables and graphs of the relationship?	4.1 Vitruvian Man: Relating Body Measurements  Focus Question: If you have data relating two variables, how can you check to see whether a linear model is a good fit?	5.1 Wood or Steel? That's the Question  Focus Question: What does a two-way table show you about preferences among groups?
1.2 Bridge Length and Strength  Focus Question: How would you describe the pattern relating bridge strength to bridge length shown in your experimental data?	2.2 Up and Down the Staircase: Exploring Slope  Focus Question: How do you write an equation for a linear function if you are given a graph, a table, or two points?	3.2 Distance, Speed and Time  Focus Question: What examples using distance, rate, and time show one variable inversely related to another?	4.2 Older and Faster: Negative Correlations  Focus Question: From the scatter plot, how do you know if a linear model fits the data? How do you know if there are outliers? How do you know if the relationship is negative or positive?	5.2 Politics of Girls and Boys: Analyzing Data in Two-Way Tables  Focus Question: Suppose you have recorded the counts of different preferences by group in a two-way table. How can you use those counts, or percents from the

				counts, to decide if two groups have the same preferences or not?
1.3 Custom Construction Parts: Finding Patterns  Focus Question: How can you predict if a pattern between variables will be linear or nonlinear?	2.3 Tree Top Fun: Equations for Linear Functions  Focus Question: What strategies do you use in writing equations for linear functions?	3.3 Planning a Field Trip: Finding Individual Cost  Focus Question: How does the cost per person change if a fixed total cost is split among an increasing number of individual payers?	4.3 Correlation Coefficients and Outliers  Focus Question: What does a correlation coefficient of 1, 0, or -1 suggest to you about the relationship between two variables?	5.3 After-School Jobs and Homework: Working Backward: Setting up a Two-Way Table  Focus Question: Suppose you have data about the same trait in two groups. How can you organize the data to compare and decide if the groups are the same or not relative to the trait?
	2.4 Boat Rental Business: Solving Linear Equations  Focus Question: What strategies do you find useful to find solutions for linear equations?	3.4 Modeling Data Patterns  Focus Question: What pattern in a table or graph of data suggests an inverse variation model and what strategies can you use to find an equation model for that kind of function?	4.4 Measuring Variability: Standard Deviation  Focus Question: How do you calculate the standard deviation for a data distribution and what does that statistic tell about the distribution?	
	2.5 Amusement Park or Movies: Intersecting			

	Linear Models  Focus Question: When the graphs of two linear functions intersect, what do the coordinates of that intersection point tell you?			
Mathematical	Mathematical	Mathematical	Mathematical	Mathematical
Reflection:	Reflection:	Reflection:	Reflection:	Reflection:
1. You can represent a	1. Why is it helpful to	1. Suppose the	1. Think about the	1. What are categorical
relationship between	use a linear model for a	relationship between	pattern of points you	variables and what do
variables with a table, a	set of data?	variables x and y is an	see in a scatter plot.	they measure?
graph, a description in		inverse variation.		
words, or an equation.	2. When does it make		a. What pattern would	2. Suppose a survey
	sense to choose a linear	a. How do the values of	you expect when two	asked teenagers and
a. How can you decide	function to model a set	y change as the values	variables are related by	adults whether or not
whether a relationship is linear by studying	of data?	of x increase?	a linear model with positive slope?	the use text messaging.
the pattern in a data	3. How would you find	b. Describe the trend in	positive slope:	a. How could you
table?	the equation for a	a graph of (x, y) values.	b. What pattern would	arrange the data to
	linear function in the	a graph of (x, y) values.	you expect when two	compare the groups?
b. How can you decide	following situations?	c. Describe the	variables are related by	gompar e erre groups.
whether a relationship		equation that relates	a linear model with	b. How would you
is linear by studying	a. You are given a	the values of x and y.	negative slope?	decide that the two
the pattern in a graph?	description of the		_	groups - teenagers and
	variables in words.	2. How is an inverse	c. What would you	adults - were different
c. How can you decide		variation similar to a	expect to see in a	in their use of text
whether a relationship	b. You are given a table	linear relationship?	scatter plot when two	messaging?
is linear by studying	of values for the	How is it different?	variables are	
the words used to	variables		unrelated?	c. Suppose that one
describe the variables?	<u> </u>			analysis compared only

d. How can you decide whether a relationship is linear by studying the equation that expresses the relationship in symbolic form?  2. What are the advantages and disadvantages in finding patterns and making predictions?	c. You are given a graph of sample data points  4. What strategies can you use to solve a linear equation such as $500 = 245 + 5x$ ?  5. What kind of mathematical sentences express "at least" and "at most" questions about linear functions?	<ul> <li>2. You assessed the accuracy of linear models.</li> <li>a. What do outliers on a scatter plot indicate?</li> <li>b. What can you learn from the errors of prediction or residuals?</li> <li>c. What do you know about a linear model from the correlation coefficient?</li> <li>3. What does the standard deviation tell</li> </ul>	the numbers in each group - teenage text messager, teenage nontext messager, adult text messager, and adult non-text messager. How might the analysis result in misleading conclusions?