7-1: Shapes and Designs

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Properties of Polygons Understand the properties of polygons that affect their shape

- Explore the ways that polygons are sorted into families according to the number and length of their sides and the size of their angles
- Explore the patterns among interior and exterior angles of a polygon
- Explore the patterns among side lengths in a polygon
- Investigate the symmetries of a shape—rotation or Reflections
- Determine which polygons fit together to cover a flat surface and why
- Reason about and solve problems involving various polygons

Relationships Among Angles Understand special relationships among angles

- Investigate techniques for estimating and measuring angles
- Use tools to sketch angles
- Reason about the properties of angles formed by parallel lines and transversals
- Use information about supplementary, complementary, vertical, and adjacent angles in a shape to solve for an unknown angle in a multi-step problem

Constructing Polygons Understand the properties needed to construct polygons

- Draw or sketch polygons with given conditions by using various tools and techniques such as freehand, use of a ruler and protractor, and use of technology
- Determine what conditions will produce a unique polygon, more than one polygon, or no polygon, particularly triangles and quadrilaterals
- Recognize the special properties of polygons, such as angle sum, side-length relationships, and symmetry, that make them useful in building, design, and nature
- Solve problems that involve properties of shapes

7-1 Shapes and Designs: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 The Family of Polygons	**Investigation 2** Designing Polygons: The Angle Connection	**Investigation 3** Designing Triangles and Quadrilaterals
Problem 1.1 **Sorting and Sketching Polygons** FQ: What properties do all polygons share? What properties do some sub-groups of polygons share? **Problem 1.2** **In a Spin: Angles and Rotations** FQ: What are some common benchmark angles? What part of a full turn is each angle equal to? **Problem 1.3** **Estimating Measures of Rotations and Angles** FQ: When a drawing shows two rays with a common endpoint, how many rotation angles are there? How would you estimate the measure of each angle? **Problem 1.4** **Measuring Angles** FQ: How do you measure an angle with an angle ruler and a protractor? **Problem 1.5** **Design Challenge I: Drawing With Tools—Ruler and Protractor** FQ: In a triangle, what measures of sides and angles give just enough information to draw a figure that is uniquely determined?	**Problem 2.1** **Angle Sums of Regular Polygons** FQ: What is the size of each angle and the sum of all angles in a regular polygon with n sides? **Problem 2.2** **Angle Sums of Any Polygon** FQ: What is the angle sum of any polygon with n sides? How do you know that your formula is correct? **Problem 2.3** **The Bees Do It: Polygons in Nature** FQ: Which regular polygons can be used to tile a surface without overlaps or gaps, and how do you know that your answer is correct? **Problem 2.4** **The Ins and Outs of Polygons** FQ: What is an exterior angle of a polygon, and what do you know about the measures of exterior angles?	**Problem 3.1** **Building Triangles** FQ: What combinations of three side lengths can be used to make a triangle? How many different shapes are possible for such a combination of side lengths? **Problem 3.2** **Design Challenge II: Drawing Triangles** FQ: What is the smallest number of side and angle measurements that will tell you how to draw an exact copy of any given triangle? **Problem 3.3** **Building Quadrilaterals** FQ: What combinations of side lengths can be used to make a quadrilateral? How many different shapes are possible for any such combination of side lengths? **Problem 3.4** **Parallel Lines and Transversals** FQ: When two parallel lines are cut by a transversal, what can be said about the eight angles that are formed? **Problem 3.5** **Design Challenge III: The Quadrilateral Game** FQ: How are squares, rhombuses, rectangles, and trapezoids similar? How are they different?
Mathematical Reflections 1. What are the common properties of all polygons? 2. What does the measure in degrees tell you about an angle? What are some common benchmark angles? 3. What strategies can be used to estimate angle measures? To deduce angle measures from given information? To find accurate measurements with tools?	**Mathematical Reflections** 1. How is the number of sides related to the sum of the interior angles in a polygon? What about the sum of the exterior angles? 2. How is the measure of each interior angle related to the number of sides in a regular polygon? What about the measure of each exterior angle? 3. Which polygons can be used to tile a flat surface without overlaps or gaps? Why are those the only figures that work as tiles?	**Mathematical Reflections** 1. What information about combinations of angle sizes and side lengths provide enough information to copy a given triangle exactly? A quadrilateral? 2. Why are triangles so useful in building structures? What are the problems with quadrilaterals for building structures? 3. If two parallel lines are intersected by a transversal, which pairs of angles will have the same measure? 4. What does it mean to say a figure has symmetry? Provide examples with your explanation.

7-2: Accentuate the Negative

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Rational Numbers Develop an understanding that rational numbers consist of positive numbers, negative numbers, and zero

- Explore relationships between positive and negative numbers by modeling them on a number line
- Use appropriate notation to indicate positive and negative numbers
- Compare and order positive and negative rational numbers (integers, fractions, decimals, and zero) and locate them on a number line
- Recognize and use the relationship between a number and its opposite (additive inverse) to solve problems
- Relate direction and distance to the number line
- Use models and rational numbers to represent and solve problems

Operations With Rational Numbers Develop understanding of operations with rational numbers and their properties

- Develop and use different models (number line, chip model) for representing addition, subtraction, multiplication, and division
- Develop algorithms for adding, subtracting, multiplying, and dividing integers
- Recognize situations in which one or more operations of rational numbers are needed
- Interpret and write mathematical sentences to show relationships and solve problems
- Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations
- Use parentheses and the Order of Operations in computations
- Understand and use the Commutative Property for addition and multiplication
- Apply the Distributive Property to simplify expressions and solve problems

7-2 Accentuate the Negative: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Extending the Number System	**Investigation 2** Adding and Subtracting Rational Numbers	**Investigation 3** Multiplying and Dividing Rational Numbers	**Investigation 4** Properties of Operations
Problem 1.1 **Playing Math Fever: Using Positive and Negative Numbers** FQ: How can you find the total value of a combination of positive and negative integers? **Problem 1.2** **Extending the Number Line** FQ: How can you use a number line to compare two numbers? **Problem 1.3** **From Sauna to Snowbank: Using a Number Line** FQ: How can you write a number sentence to represent a change on a number line, and how can you use a number line to represent a number sentence? **Problem 1.4** **In the Chips: Using a Chip Model** FQ: How can you use a chip model to represent addition and subtraction?	**Problem 2.1** **Extending Addition to Rational Numbers** FQ: How can you predict whether the result of addition of two numbers will be positive, negative, or zero? **Problem 2.2** **Extending Subtraction to Rational Numbers** FQ: How is a chip model or number line useful in determining an algorithm for subtraction? **Problem 2.3** **The "+/-" Connection** FQ: How are the algorithms for addition and subtraction of integers related? **Problem 2.4** **Fact Families** FQ: What related sentence is equivalent to $4 + n = 43$ and makes it easier to find the value of n?	**Problem 3.1** **Multiplication Patterns With Integers** FQ: How is multiplication of two integers represented on a number line and chip board? **Problem 3.2** **Multiplication of Rational Integers** FQ: What algorithm can you use for multiplying integers? **Problem 3.3** **Division of Rational Numbers** FQ: What algorithm can you use for dividing integers? How are multiplication and division related? **Problem 3.4** **Playing the Integer Product Game: Applying Multiplication and Division of Integers** FQ: What patterns do you notice on the game board for the Integer Product Game that can help you?	**Problem 4.1** **Order of Operations** FQ: Does the Order of Operations work for integers? Explain. **Problem 4.2** **The Distributive Property** FQ: How can you use the Distributive Property to expand an expression or factor an expression that involves integers? **Problem 4.3** **What Operations are Needed?** FQ: What information in a problem is useful to help you decide which operation to use to solve the problem?
Mathematical Reflections 1. How do decide which of two numbers is greater when 1a. both numbers are positive? 1b. both numbers are negative? 1c. one number is positive and one number is negative? 2. How does a number line help you compare numbers? 3. When you add a positive number and a negative number, how do you determine the sign of the answer? 4. If you are doing a subtraction problem on a chip board, and the board does not have enough chips of the color you wish to subtract, what can you do to make the subtraction possible?	**Mathematical Reflections** 1a. What algorithm(s) will produce the correct result for the sum "$a + b$," where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works. 1b. What algorithm(s) will produce the correct result for the difference "$a – b$," where a and b each represent any rational number? Show, using a number line or chip board, why your algorithm works. 2. How can any difference "$a – b$" be restated as an equivalent addition statement, where a and b each represent any rational number? 3a. What does it mean to say that an operation is *commutative*? 3b. Describe some ways that the additive inverse of a number is important.	**Mathematical Reflections** 1. Give an example of a multiplication problem, involving two integers, in which the product is 1a. less than 0. 1b. greater than 0. 1c. equal to 0. 1d. In general, describe the signs of the factors for each product in parts (a)–(c). 2. Give an example of a division problem, involving two integers, in which the quotient is 2a. less than 0. 2b. Greater than 0. 2c. Equal to 0. 2d. In general, describe the signs of the dividend and divisor for each quotient in parts (a)–(c). 3a. Suppose three numbers are related by an equation of the form $a \cdot b = c$, where a, b, and c are not equal to 0. Write two related number sentences using multiplication. 3b. Suppose three numbers are related by an equation of the form $a \div b = c$, where a, b, and c are not equal to 0. Write two related number sentences using multiplication. 4. Which operations on integers are commutative? Give numerical examples to support your answer.	**Mathematical Reflections** 1a. What is the Order of Operations? Why is the Order of Operations important? 1b. Give an example of a numerical expression in which the use of parentheses changes the result of the computation. 2. Describe how the Distributive Property relates addition and multiplication. Give numerical examples.

7-3: Stretching and Shrinking

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Similar Figures Understand what it means for figures to be similar

- Identify similar figures by comparing corresponding sides and angles
- Use scale factors and ratios to describe relationships among the side lengths, perimeters, and areas of similar figures
- Generalize properties of similar figures
- Recognize the role multiplication plays in similarity relationships
- Recognize the relationship between scale factor and ratio in similar figures
- Use informal methods, scale factors, and geometric tools to construct similar figures (scale drawings)
- Compare similar figures with nonsimilar figures
- Distinguish algebraic rules that produce similar figures from those that produce nonsimilar figures
- Use algebraic rules to produce similar figures
- Recognize when a rule shrinks or enlarges a figure
- Explore the effect on the image of a figure if a number is added to the x- or y-coordinates of the figure's vertices

Reasoning with Similar Figures Develop strategies for using similar figures to solve problems

- Use the properties of similarity to find distances and heights that cannot be measured directly
- Predict the ways that stretching or shrinking a figure will affect side lengths, angle measures, perimeters, and areas
- Use scale factors or ratios to find missing side lengths in a pair of similar figures
- Use similarity to solve real-world problems

7-3 Stretching and Shrinking: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Enlarging and Reducing Shapes	**Investigation 2** Similar Figures	**Investigation 3** Scaling Perimeter and Area	**Investigation 4** Similarity and Ratios
Problem 1.1 **Solving a Mystery: An Introduction to Similarity** FQ: What does it mean for two figures to be similar? **Problem 1.2** **Scaling Up and Down: Corresponding Sides and Angles** FQ: When you copy a figure at a certain scale factor (e.g. 150%), how does this value affect the measurements of the new figure?	**Problem 2.1** **Drawing Wumps: Making Similar Figures** FQ: How can you determine if two shapes are similar by looking at the rule for producing specific coordinates for the image? **Problem 2.2** **Hats Off to the Wumps: Changing a Figure's Size and Location** FQ: What types of coordinate rules produce similar figures? Nonsimilar figures? For a pair of similar figures, how can you use a coordinate rule to predict the side lengths of the image? **Problem 2.3** **Mouthing Off and Nosing Around: Scale Factors** FQ: How can you decide whether or not two shapes are similar?	**Problem 3.1** **Rep-Tile Quadrilaterals: Forming Rep-Tiles With Similar Quadrilaterals** FQ: What types of quadrilaterals are rep-tiles? How do rep-tiles show that the scale factors and areas of similar quadrilaterals are related? **Problem 3.2** **Rep-Tile Triangles: Forming Rep-Tiles With Similar Figures** FQ: Which types of triangles are rep-tiles? Explain. **Problem 3.3** **Designing Under Constraints: Scale Factors and Similar Shapes** FQ: How can you use scale factors to draw similar figures or to find missing side lengths in similar figures? **Problem 3.4** **Out of Reach: Finding Lengths with Similar Triangles** FQ: How can you use similar triangles to find a distance that is difficult to measure directly?	**Problem 4.1** **Ratios Within Similar Parallelograms** FQ: What information does the ratio of adjacent side lengths within a rectangle give you? **Problem 4.2** **Ratios Within Similar Triangles** FQ: For a pair of triangles, if the measures of corresponding angles are equal, how can you use ratios of side lengths to determine whether or not the triangles are similar? **Problem 4.3** **Finding Missing Parts: Using Similarity to Find Measurements** FQ: If two shapes are similar, how can you use information about the shapes to find unknown side lengths, perimeters, and areas? **Problem 4.4** **Using Shadows to Find Heights: Using Similar Triangles** FQ: How can you use similar triangles to estimate the heights of tall objects?
Mathematical Reflections 1a. When you enlarge or reduce a figure, what features stay the same? 1b. When you enlarge or reduce a figure, what features change? 2. Rubber-band stretchers, copy machines, and projectors all make images that are similar to the original shapes. What does it mean for two shapes to be similar? Complete the sentence below: *"Two geometric shapes are similar when…"*	**Mathematical Reflections** 1. If two shapes are similar, what is the same about them and what is different? 2a. What does the scale factor tell you about two similar figures? 2b. How does the coordinate rule for making two similar shapes relate to the scale factor? 3. Rubber band stretchers, copy machines, and coordinate grids all made images that are similar to (or scale drawings of) the original shapes. What does it mean to say two shapes are similar? Build on your statement from Mathematical Reflections 1: *"Two geometric shapes are similar when…"*	**Mathematical Reflections** 1a. If two polygons are similar, how can find the scale factor from one polygon to the other? Give specific examples. 1b. Suppose you are given a polygon. How can you draw a similar figure? 2. What does the scale factor between two similar figures tell you about the 2a. side lengths? 2b. perimeters? 2c. areas? 2d. angles? 3. If two figures are similar, how can you find a missing side length? 4. Describe how you can find the measure of a distance that you cannot measure directly. 5. What does it mean to say two shapes are similar? After completing Investigation 3, how can you build on your statements from Mathematical Reflections 1 and 2? *"Two geometric shapes are similar when…"*	**Mathematical Reflections** 1. If two triangles, rectangles, or parallelograms are similar, 1a. How does the ratio of two side lengths within one figure compare to the ratio of the corresponding side lengths in the other figure? 1b. What does the scale factor from one figure to the other tell you about the figures? 2a. Describe at least two ways to find a missing side length in a pair of similar figures. 2b. How can you find the height of an object that cannot be measured directly? 3. What does it mean to say that two shapes are similar? After exploring with ratios, build on your statements from Mathematical Reflections 1, 3, and 3: *"Two geometric shapes are similar when…*

7-4: Comparing and Scaling

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Ratios, Rates, and Percents Understand ratios, rates, and percents

- Use ratios, rates, fractions, differences, and percents to write statements comparing two quantities in a given situation
- Distinguish between and use both part-to-part and part-to-whole ratios in comparisons
- Use percents to express ratios and proportions
- Recognize that a rate is a special ratio that compares two measurements with different units
- Analyze comparison statements made about quantitative data for correctness and quality
- Make judgments about which kind of comparison statements are most informative or best reflect a particular point of view in a specific situation

Proportionality Understand proportionality in tables, graphs, and equations

- Recognize that constant growth in a table, graph, or equation is related to proportional situations
- Write an equation to represent the pattern in a table or graph of proportionally related variables
- Relate the unit rate and constant of proportionality to an equation, graph, or table describing a proportional situation

Reasoning Proportionally Develop and use strategies for solving problems that require proportional reasoning

- Recognize situations in which proportional reasoning is appropriate to solve the problem
- Scale a ratio, rate, percent, or fraction to make a comparison or find an equivalent representation
- Use various strategies to solve for an unknown in a proportion, including scaling, rate tables, percent bars, unit rates, and equivalent ratios
- Set up and solve proportions that arise from real-world applications, such as finding discounts and markups and converting measurement units

7-4 Comparing and Scaling: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Ways of Comparing: Ratios and Proportions	**Investigation 2** Comparing and Scaling Rates	**Investigation 3** Markups, Markdowns, and Measures: Using Ratios, Percents, and Proportions
Problem 1.1 **Surveying Opinions: Analyzing Comparison Statements** FQ: What do different comparisons of quantities tell you about their relationship? **Problem 1.2** **Mixing Juice: Comparing Ratios** FQ: What strategies do you use to determine which mix is the most orangey? **Problem 1.3** **Time to Concentrate: Scaling Ratios** FQ: When you scale up a recipe and change the units, like from cups to ounces, what are some of the issues you have to deal with? **Problem 1.4** **Keeping Things in Proportion: Scaling to Solve Proportions** FQ: What strategies can you use to find a missing value in a proportion? What is your preferred strategy and why?	**Problem 2.1** **Sharing Pizza: Comparison Strategies** FQ: How can you determine whether two ratios are equivalent or find which of two ratios is more favorable? **Problem 2.2** **Comparing Pizza Prices: Scaling Rates** FQ: How can you use rate tables to find missing values? How are rate tables similar to scaling quantities and solving proportions? **Problem 2.3** **Finding Costs: Unit Rate and Constant of Proportionality** FQ: How can you find a unit rate in a description, an equation, a table, or a graph?	**Problem 3.1** **Commissions, Markups, and Discounts: Proportions With Percents** FQ: How can you use proportions and percent tables to find various percentages of a value when you know a certain percentage of the same value? **Problem 3.2** **Measuring to the Unit: Measurement Conversions** FQ: How can you use unit rates, proportions, equations, and rate tables to scale a variety of units? **Problem 3.3** **Mixing it Up: Connecting Ratios, Rates, Percents, and Proportions** FQ: How can you use scale factors, rate tables, proportions, equations, or graphs to find amounts of a mixture, given the proportions?
Mathematical Reflections 1a. In this Investigation you have used ratios, percents, fractions, and differences to make comparison statements. How have you found these ideas helpful? 1b. Give examples to explain how part-to-part ratios are different from, but related to, part-to-whole ratios. 2. How can you use scaling or equivalent rations 2a. to solve a proportion? Give an example. 2b. To make a decision? Give an example. 3. You learned about scaling in *Stretching and Shrinking*. You learned about proportions and rates in *Comparing and Scaling*. How are the ideas in these two Units the same? How are they different? 4. Describe the connections you have found among unit rates, proportions, and rate tables.	**Mathematical Reflections** 1a. How are tables, graphs, and equations helpful when you work with proportions? 1b. How can you identify a unit rate or constant of proportionality in a table? In a graph? In an equation? 2. How are unit rates useful? 3. How is finding a unit rate similar to solving a proportion?	**Mathematical Reflections** 1. What strategies have you learned for solving proportions? 2. Describe a strategy for converting a rate measured in one pair of units to a rate measured in a different pair of units. For example, how would you convert ounces per cup to pounds per gallon? 3. You learned about scaling in *Stretching and Shrinking*. You learned about proportions and rates in *Comparing and Scaling*. How are the ideas in these two Units the same? How are they different? 4. Describe the connections you have found among unit rates, proportions, and rate tables.

7-5: Moving Straight Ahead

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear Relationships Recognize problem situations in which two variables have a linear relationship

- Identify and describe the patterns of change between the independent and dependent variables for linear relationships represented by tables, graphs, equations, or contextual settings
- Construct tables, graphs, and symbolic equations that represent linear relationships
- Identify the rate of change between two variables and the x- and y-intercepts from graphs, tables, and equations that represent linear relationships
- Translate information about linear relationships given in a contextual setting, a table, a graph, or an equation to one of the other forms
- Write equations that represent linear relationships given specific pieces of information, and describe what information the variables and numbers represent
- Make a connection between slope as a ratio of vertical distance to horizontal distance between two points on a line and the rate of change between two variables that have a linear relationship
- Recognize that $y=mx$ represents a proportional relationship
- Solve problems and make decisions about linear relationships using information given in tables, graphs, and equations

Equivalence Understand that the equality sign indicates that two expressions are equivalent

- Recognize that the equation $y=mx+b$ represents a linear relationship and means that $mx+b$ is an expression equivalent to y
- Recognize that linear equations in one unknown, $k=mx+b$ or $y=m(t)+b$, where k, t, m, and b are constant numbers, are special cases of the equation $y=mx+b$
- Recognize that finding the missing value of one of the variables in a linear relationship, $y=mx+b$, is the same as finding a missing coordinate of a point (x,y) that lies on the graph of the relationship
- Solve linear equations in one variable using symbolic methods, tables, and graphs
- Recognize that a linear inequality in one unknown is associated with a linear equation
- Solve linear inequalities using graphs or symbolic reasoning
- Show that two expressions are equivalent
- Write and interpret equivalent expressions

7-5 Moving Straight Ahead: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Walking Rates	**Investigation 2** Exploring Linear Relationships with Graphs and Tables	**Investigation 3** Solving Equations	**Investigation 4** Exploring Slope: Connecting Rates and Ratios
Problem 1.1 **Walking Marathons: Finding and Using Rates** FQ: What equation represents the relationship between the time and the distance you walk at a constant rate? What are the dependent and independent variables? **Problem 1.2** **Walking Rates and Linear Relationships: Tables, Graphs, and Equations** FQ: How can you predict whether a relationship is linear from a table, a graph, or an equation that represents the relationship? **Problem 1.3** **Raising Money: Using Linear Relationships** FQ: What is the pattern of change in a linear relationship? **Problem 1.4** **Using the Walkathon Money: Recognizing Linear Relationships** FQ: How can you determine if a linear relationship is increasing or decreasing?	**Problem 2.1** **Henri and Emile's Race: Finding the Point of Intersection** FQ: When is it helpful to use a graph or table to solve a problem? **Problem 2.2** **Crossing the Line: Using Tables, Graphs, and Equations** FQ: How does the pattern of change for a linear relationship appear in a table, a graph, or an equation? **Problem 2.3** **Comparing Costs: Comparing Relationships** FQ: How can you decide if a table or an equation represents a linear relationship? **Problem 2.4** **Connecting Tables, Graphs, and Equations** FQ: How are solutions of an equation of the form y = b + mx related to the graph and the table for the same relationship?	**Problem 3.1** **Solving Equations Using Tables and Graphs** FQ: How are the coordinates of a point on a line or in a table related to the equation of the line? **Problem 3.2** **Mystery Pouches in the Kingdom of Montarek: Exploring Equality** FQ: What does equality mean? **Problem 3.3** **From Pouches to Variables: Writing Equations** FQ: How can the properties of equality be used to solve linear equations? **Problem 3.4** **Solving Linear Equations** FQ: What are some strategies for solving linear equations? **Problem 3.5** **Finding the Point of Intersection: Equations and Inequalities** FQ: How can you find when two expressions are equal, or when one expression is greater or less than the other?	**Problem 4.1** **Climbing Stairs: Using Rise and Run** FQ: How is the steepness of a set of stairs related to a straight-line graph? **Problem 4.2** **Finding the Slope of a Line** FQ: How can you find the y-intercept and the slope of a line from data in a table, graph, or equation? **Problem 4.3** **Exploring Patterns with Lines** FQ: How can you predict if two lines are parallel or perpendicular from their equations? **Problem 4.4** **Pulling it All Together: Writing Equations for Linear Relationships** FQ: What information do you need to write an equation for a linear relationship? Is the expression for the dependent variable always the same?
Mathematical Reflections 1. Describe how the dependent variable changes as the independent variable changes in a linear relationship. Give examples. 2. How does the pattern of change between two variables in a linear relationship show up in 2a. a contextual situation? 2b. a table? 2c. a graph? 2d. an equation?	**Mathematical Reflections** 1a. Explain how the information about a linear relationship is represented in a table, a graph, or an equation. 1b. Describe several real-world situations that can be modeled by equations of the form y = mx + b and y = mx. Explain how the latter equation represents a proportional relationship. 2a. Explain how a table or graph that represent a linear relationship can be used to solve a problem. 2b. Explain how you have used an equation that represents a linear relationship to solve a problem.	**Mathematical Reflections** 1a. Suppose that, in an equation with two variables, you know the value of one of the variables. Describe a method for finding the value of the other variable using the properties of equality. Give an example to illustrate your method. 1b. Compare the method you described in part (a) to the methods of using a table or a graph to solve linear equations. 2a. Explain how an inequality can be solved by methods similar to those used to solve linear equations. 2b. Describe a method for finding the solution to an inequality using graphs. 3. Give an example of two equivalent expressions that were used in this investigation. Explain why they are equivalent.	**Mathematical Reflections** 1. Explain what the slope of a line is. How does finding the slope compare to finding the rate of change between two variables in a linear relationship? 2. How can you find the slope of a line from 2a. an equation? 2b. a graph? 2c. a table of values of the line? 2d. the coordinates of two points on the line? 3. For parts (a) and (b), explain how you can write an equation of a line from the information. Use examples to illustrate your thinking. 3a. the slope and the y-intercept of the line 3b. two points on the line

7-6 What Do You Expect

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Experimental and Theoretical Probabilities Understand experimental and theoretical probabilities

- Recognize that probabilities are useful for predicting what will happen over the long run
- For an event described in everyday language, identify the outcomes in a sample space that compose the event
- Interpret experimental and theoretical probabilities and the relationship between them and recognize that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers
- Distinguish between outcomes that are equally likely or not equally likely by collecting data and analyzing experimental probabilities
- Realize that the probability of simple events is a ratio of favorable outcomes to all outcomes in the sample space
- Recognize that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
- Determine the fairness of a game

Reasoning With Probability Explore and develop probability models by identifying possible outcomes and analyze probabilities to solve problems

- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process
- Represent sample spaces for simple and compound events and find probabilities using organized lists, tables, tree diagrams, area models, and simulation
- Realize that, just as with simple events, the probability of a compound event is a ratio of favorable outcomes to all outcomes in the sample space
- Design and use a simulation to generate frequencies for simple and compound events
- Analyze situations that involve two or more stages (or actions) called *compound events*
- Use area models to analyze the theoretical probabilities for two-stage outcomes
- Analyze situations that involve binomial outcomes
- Use probability to calculate the long-term average of a game of chance
- Determine the expected value of a probability situation
- Use probability and expected value to make a decision

7-6 What Do You Expect: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 A First Look at Chance	**Investigation 2** Experimental and Theoretical Probability	**Investigation 3** Making Decisions With Probability	**Investigation 4** Analyzing Compound Events Using an Area Model	**Investigation 5** Binomial Outcomes
Problem 1.1 **Choosing Cereal: Tossing a Coin to Find Probabilities** FQ: How does collecting more data help you predict the outcome of a situation? **Problem 1.2** **Tossing Paper Cups: Finding More Probabilities** FQ: How does modeling with an experiment help you determine possible outcomes and the likelihood of each outcome? **Problem 1.3** **One More Try: Finding Experimental Probabilities** FQ: How do you determine the relative frequency of an outcome? **Problem 1.4** **Analyzing Events: Understanding Equally Likely** FQ: How can you determine whether the outcomes of a probability event are all equally likely, and why would this information matter?	**Problem 2.1** **Predicting to Win: Finding Theoretical Probabilities** FQ: How does experimental probability compare to theoretical probability for a given situation? **Problem 2.2** **Choosing Marbles: Developing Probability Models** FQ: What are some properties of theoretical probabilities? **Problem 2.3** **Designing a Fair Game: Pondering Possible and Probable** FQ: How can you decide whether a game is fair or not? **Problem 2.4** **Winning the Bonus Prize: Using Strategies to Find Theoretical Probabilities** FQ: How can you determine all of the probabilities for a compound event?	**Problem 3.1** **Designing a Spinner to Find Probabilities** FQ: How do you determine probability using a spinner? **Problem 3.2** **Making Decisions: Analyzing Fairness** FQ: When using a tool to simulate a fair game, what things must you consider? **Problem 3.3** **Roller Derby: Analyzing a Game** FQ: How does understanding probability help you design a winning strategy? **Problem 3.4** **Scratching Spots: Designing and Using a Simulation** FQ: How can you design a simulation to determine probability?	**Problem 4.1** **Drawing Area Models to Find the Sample Space** FQ: How can an area model represent a situation to help analyze probabilities? **Problem 4.2** **Making Purple: Area Models and Probability** FQ: How can you use experimental or theoretical probabilities of a compound event to predict the number of times one particular combination will occur out of any given number of repetitions of the event? **Problem 4.3** **One-and-One Free Throws: Simulating a Probability Situation** FQ: How is an area model for the one-and-one free-throw situation like or unlike the area model for the Making Purple game? **Problem 4.4** **Finding Expected Value** FQ: How is expected value different from probabilities of outcomes?	**Problem 5.1** **Guessing Answers: Finding More Expected Values** FQ: If you do not know the answers to a true/false test, what is the probability that you can get a good score with random guesses? **Problem 5.2** **Ortonville Binomial Probability** FQ: What patterns are there in models for binomial probability situations that are equally likely? How do these patterns help you answer probability questions? **Problem 5.3** **A Baseball Series: Expanding Binomial Probability** FQ: If two teams are evenly matched, how do binomial probabilities help you figure out the probabilities that a winner of the required number of games will occur after a certain number of games?
Mathematical Reflections 1. How do you find the experimental probability that a particular result will occur? Why is it called the experimental probability? 2. In an experiment, are 30 trials as good as 500 trials to predict the chances of a result? Explain/ 3. What does it mean for results to be equally likely?	**Mathematical Reflections** 1. Describe how you can find the theoretical probability of an outcome. Why is it called theoretical probability? 2a. Suppose two people do an experiment to estimate the probability of an outcome. Will they get the same probabilities? Explain. 2b. Two people analyze a situation to find the theoretical probability of an outcome. Will they get the same probabilities? Explain. 2c. One person uses an experiment to estimate the probability of an outcome. Another person analyzes the situation to find the theoretical probability of the outcome. Will they get the same probabilities? Explain. 3. What does it mean for a game to be fair? 4. What is a sample space, and how can it be represented?	**Mathematical Reflections** 1. Describe a situation in which you and a friend can use probability to make a decision. Can the probabilities of the outcomes be determined both experimentally and theoretically? Why or why not? 2. Describe a situation in which it is difficult or impossible to find the theoretical probabilities of the outcomes. 3. Explain what it means of a probability situation to be fair. 4. Describe some of the strategies for determining the theoretical probabilities for situations in this unit. Give an example of a situation for each of the strategies.	**Mathematical Reflections** 1. Describe four probability situations that involve two actions. Describe the outcomes for these situations. 2. You can use an area model or a simulation to determine the probability of a situation that involves two actions. Explain how each of these is used. 3. Describe how you would calculate the expected value for a probability situation. 4. Expected value is sometimes called the longer-term average. Explain why this makes sense.	**Mathematical Reflections** 1. Describe five different binomial situations. Explain why they are binomial situations. 2. Tossing a coin three times is an example of a situation involving a series of three actions, each with two equally likely outcomes. 2a. Pick one of the situation in Question 1. Describe a series of three actions, each with two equally likely outcomes. Make a list of all the possible outcomes. 2b. Write a question about your situation that can be answered by your list. 3. As you increase the number of actions for a binomial situation, what happens to the total number of possible outcomes? For example, suppose you increase the number of times a coin is tossed. What happens to the total number of outcomes?

7-7: Filling and Wrapping

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Surface Areas and Volumes of Polygonal Prisms and Cylinders Understand surface areas and volumes of prisms and cylinders and how they are related

- Describe prisms by using their vertices, faces, and edges
- Visualize three-dimensional shapes and the effects of slicing those shapes by planes
- Deepen understanding of volumes and surface areas of rectangular prisms
- Estimate and calculate surface areas and volumes of polygonal prisms by relating them to rectangular prisms
- Explore the relationships between the surface areas and volumes of prisms
- Relate surface areas and volumes for common figures, especially optimization of surface area for fixed volume
- Predict the effects of scaling dimensions on linear, surface area, and volume measures of prisms, cylinders, and other figures
- Investigate the relationship between volumes of prisms and volumes of cylinders as well as the relationship between surface areas of prisms and surface areas of cylinders
- Use volumes and surface areas of prisms to develop formulas for volumes and surface areas of cylinders
- Discover that volumes of prisms and cylinders can be calculated as the product of the area of the base and the height
- Solve problems involving surface areas and volumes of solid figures

Areas and Circumferences of Circles Understand the areas and circumferences of circles and how they are related

- Relate area of a circle to covering a figure and circumference to surrounding a figure
- Estimate and calculate areas and circumferences of circles
- Explore the relationship between circle radius (or diameter) and circumference
- Explore the relationship between circle radius (or diameter) and area
- Investigate the connection of π to area calculation by estimating the number of radius squares needed to cover a circle
- Investigate the relationship between area and circumference of a circle
- Solve problems involving areas and circumferences of circles

Volumes of Spheres and Cones Understand the relationships between the volumes of cylinders and the volumes of cones and spheres

- Relate volumes of cylinders to volumes of cones and spheres
- Estimate and calculate volumes of spheres and cones
- Solve problems involving surface areas and volumes of spheres and cone.

7-7 Filling and Wrapping: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Building Smart Boxes: Rectangular Prisms	**Investigation 2** Polygonal Prisms	**Investigation 3** Area and Circumference of Circles	**Investigation 4** Cylinders, Cones, and Spheres
Problem 1.1 **How Big Are Those Boxes? Finding Volume** FQ: How do you calculate the surface area and volume of a rectangular prism? **Problem 1.2** **Optimal Containers I: Finding Surface Area** FQ: Suppose you design a box in the shape of a rectangular prism with a volume of 24 cm^3. What are the shape and dimensions of the box that has minimum surface area? **Problem 1.3** **Optimal Containers II: Finding the Least Surface Area** FQ: What are the dimensions of the rectangular prism that has the least surface area for a given volume? **Problem 1.4** **Compost Containers: Scaling Up Prisms** FQ: As you change the dimensions of a rectangular prism by a certain scale factor, how do the surface area and volume of the prism change?	**Problem 2.1** **Folding Paper: Surface Area and Volume of Prisms** FQ: For a prism with fixed height and fixed lateral area, how do the volume and surface area of the prism change as the number of sides increases? **Problem 2.2** **Packing a Prism: Calculating Volume of Prisms** FQ: What general strategy can be used to find the volume of any prism—triangular, rectangular, pentagonal, and so on? **Problem 2.3** **Slicing Prisms and Pyramids** FQ: What surface shapes and three-dimensional figures can be created by slicing a rectangular prism in various directions?	**Problem 3.1** **Going Around in Circles: Circumference** FQ: What is the relationship between the diameter or radius of a circle and its circumference? **Problem 3.2** Pricing Pizza: Connecting Area, Diameter, and Radius FQ: How does the area of a circle increase as the circle's radius and diameter increase? **Problem 3.3** **Squaring a Circle to Find is Area** FQ: What is the relationship between the area of a circle and its radius? **Problem 3.4** **Connecting Circumference and Area** FQ: What is the relationship between the circumference and area of a circle?	**Problem 4.1** **Networking: Surface Area of Cylinders** FQ: How can you calculate the surface area of a cylinder? Why does that strategy work? **Problem 4.2** **Wrapping Paper: Volume of Cylinders** FQ: How can you calculate the volume of a cylinder? How is the procedure similar to calculating the volume of a prism? **Problem 4.3** **Comparing Juice Containers: Comparing Surface Areas** FQ: How does the surface area of a cylinder compare to the surface area of a rectangular prism for a given volume? **Problem 4.4** **Filling Cones and Spheres** FQ: If a sphere and a cone have the same dimensions as a cylinder, how do the volumes compare? What formulas for volume of a sphere and the volume of a cone can you write using these relationships? **Problem 4.5** **Comparing Volumes of Spheres, Cylinders, and Cones** FQ: What are some relationships you can use involving a cone, a sphere, and a cylinder with the same dimensions?
Mathematical Reflections 1. How can you calculate the volume and surface area of a rectangular prism from measures of its length, width, and height? Explain why this works. 2. How are the surface area and volume of a rectangular prism related to each other? 3. How will the surface area and volume of a prism change in each of the following cases? 3a. You increase or reduce one dimension by a scale factor of *f*. 3b. You increase or reduce two dimensions by a scale factor of *f*. 3c. You increase or reduce all three dimensions by a scale factor of *f*.	**Mathematical Reflections** 1. How can you find the surface area of any right prism? Explain why your method works. 2. How can you find the volume of any right prism? Explain why your method works 3. What two- and three-dimensional shapes result when a right rectangular prism is cut by 3a. a horizontal slice? 3b. a vertical slice? 3c. a slanted slice?	**Mathematical Reflections** 1. How can you find the circumference and area of a circle from measures of its radius or diameter? 2. How is the challenge of finding circumferences and areas of circles similar to that of finding perimeters and areas of polygons such as triangles, rectangles, and other parallelograms? In what ways are those tasks different?	**Mathematical Reflections** 1a. Compare the task of finding the circumference of the base and the surface area of a cylinder to that of finding the perimeter of the base and the surface area a prism. 1b. Compare the task of finding the volume of cylinders to that of finding the volume of prisms. 1c. How can you find the circumference of the base, the surface area, and the volume of a cylinder from measures of its radius or diameter and its height? Explain why your formulas make sense. 1d. How do the surface area and the volume of a cylinder change if both the radius and height are changed by a factor of *f*? 2a. How is the task of finding the volumes of spheres and cones similar to that of finding the volumes of prisms and cylinders? In what ways are those tasks different? 2b. How can you find the volume of a sphere or a cone from measures of its dimensions?

7-8: Samples and Populations

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

The Process of Statistical Investigation Deepen the understanding of the process of statistical investigation and apply this understanding to samples

- Pose questions, collect data, analyze data, and interpret data to answer questions

Analysis of Samples Understand that data values in a sample vary and that summary statistics of samples, even same-sized samples, taken from the same population also vary

- Choose appropriate measures of center (mean, median, or mode) and spread (range, IQR, or MAD) to summarize a sample
- Choose appropriate representations to display distributions of samples
- Compare summary statistics of multiple samples drawn from either the same population or from two different populations and explain how the samples vary

Design and Use of Simulations Understand that simulations can model real-world situations

- Design a model that relies on probability concepts to obtain a desired result
- Use the randomly generated frequencies for events to draw conclusions

Predictions and Conclusions About Populations Understand that summary statistics of a representative sample can be used to gain information about a population

- Describe the benefits and drawbacks to various sampling plans
- Use random-sampling techniques to select representative samples
- Apply concepts from probability to select random samples from populations
- Explain how sample size influences the reliability of sample statistics and resulting conclusions and predictions
- Explain how different sampling plans influence the reliability of sample statistics and resulting conclusions and predictions
- Use statistics from representative samples to draw conclusions about populations
- Use measures of center, measures of spread, and data displays from more than one random sample to compare and draw conclusions about more than one population
- Use mean and MAD, or median and IQR, from random samples to assess whether the differences in the samples are due to natural variability or due to meaningful differences in the underlying populations

7-8 Samples and Populations: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Making Sense of Samples	**Investigation 2** Choosing a Sample From a Population	**Investigation 3** Using Samples to Draw Conclusions
Problem 1.1 **Comparing Performances: Using Center and Spread** FQ: Given a set of results, how might you use measures of center and variability (spread) to judge overall performance? **Problem 1.2** **Which Team Is Most Successful? Using the MAD to Compare Samples** FQ: What strategies might you use to evaluate numerical outcomes and judge success? **Problem 1.3** **Pick Your Preference: Distinguishing Categorical Data From Numerical Data** FQ: How might you compare results to see if each sample responded to a survey in a similar way? How can using percentages help you make comparisons? **Problem 1.4** **Are Steel-Frame Coasters Faster Than Wood-Frame Coasters? Using the IQR to Compare Samples** FQ: How might you decide whether steel-frame coasters or wood-frame coasters are faster?	**Problem 2.1** **Asking About Honesty: Using a Sample to Draw Conclusions** FQ: What is a population? What is a sample? What is a sampling plan? **Problem 2.2** **Selecting a Sample: Different Kinds of Samples** FQ: How could you select a sample of your school population to survey? **Problem 2.3** **Choosing Random Samples: Comparing Samples Using Center and Spread** FQ: How could you use statistics of a random sample of data to make predictions about an entire population? **Problem 2.4** **Growing Samples: What Size Sample to Use?** FQ: Can you make good statistical estimates with less work by selecting smaller samples? How does sample size relate to the accuracy of statistical estimates?	**Problem 3.1** **Solving an Archeological Mystery: Comparing Samples Using Box Plots** FQ: How might you analyze samples from known and unknown populations to determine whether the unknown population has one or more attributes in common with the known population? **Problem 3.2** **Comparing Heights of Basketball Players: Using Means and MADs** FQ: How can you determine whether differences in sample data are large enough to be meaningful, or just due to naturally occurring variability from one sample to another? **Problem 3.3** **Five Chocolate Chips in Every Cookie: Using Sampling in a Simulation** FQ: How can you simulate a real-world problem? How can you analyze the data that you collect from that simulation to draw conclusions? **Problem 3.4** **Estimating a Deer Population: Using Samples to Estimate the Size of a Population** FQ: How can you estimate the size of a large population?
Mathematical Reflections 1a. A new term is used in this Investigation: sample. What do you think sample means? 1b. Suppose you have data from a 7th-grade class. The data are answers to the questions: • What is your favorite movie? • How many movies do you watch per week? i. Which statistic can you use to summarize the results of the data? ii. How could you use the data to predict the number of students in the entire 7th grade who would say they watch two movies per week? 2a. How do graphs of distributions help you compare data sets? 2b. How do measures of center help you compare data sets? 2c. How do measures of spread help you compare data sets? 3. When does it make sense to compare groups using counts, or frequencies? When does it make sense to compare groups using percents, or relative frequencies? Explain.	**Mathematical Reflections** 1. Why are data often collected from a sample rather than from an entire population? 2. Describe four plans for selecting a sample from a population. Discuss the advantages and disadvantages of each plan. 3a. How are random samples different from convenience, voluntary-response, and systematic samples? 3b. Why is random sampling preferable to the other sampling plans? 3c. Describing three plans for selecting a random sample from a given population. What are the advantages and disadvantages of each plan? 4. Suppose you select several random samples of size 30 from the same population. 4a. When you compare the samples to each other, what similarities and differences would you expect to find among the measures of center and spread? 4b. When you compare the samples to the larger population, what similarities and differences would you expect to find among the measures of center and spread? 5. How has your idea of the term sample changed from what you wrote in Mathematical Reflections, Investigation 1?	**Mathematical Reflections** 1a. How can you use statistics to compare samples? How can you use samples to draw conclusions about the populations from which they are selected? 1b. In what ways might a data distribution for a sample be similar to or different from the data distribution for the entire population? 2a. How can you use box plots, medians, and IQRs to compare samples? Give an example. 2b. How can you use means and MADs to compare samples? Give an example. 2c. How can you use statistics to decide whether differences between samples are expected due to natural variability or reflect measureable differences in underlying populations? 3a. How can you use simulations to generate samples? 3b. How can you use data from a capture-tag-recapture simulation to estimate the actual size of a population? 4. The process of statistical investigation involves posing questions, collecting and analyzing data, and making interpretations to answer the original questions. Choose a Problem from this Investigation. Explain how you used the process of statistical investigation to solve the Problem.