

8-1: Thinking with Mathematical Models

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear and Nonlinear Relationships Recognize and model linear and nonlinear relationships in bivariate data

- Represent data patterns using graphs, tables, word descriptions and algebraic expressions
- Use mathematical models to answer questions about linear relationships
- Investigate the nature of linear variation in contexts
- Write linear functions from verbal, numerical, or graphical information
- Analyze, approximate, and solve linear equations
- Model situations with inequalities expressed as “at most” and “at least” situations
- Investigate the nature of inverse variation in contexts
- Use mathematical models to answer questions about inverse variation relationships
- Compare inverse variation relationships with linear relationships

Data Analysis Measure variation in data and strength of association in bivariate data

- Use data patterns to make predictions
- Fit a line to data that show a linear trend and measure goodness of fit
- Analyze scatter plots of bivariate data to determine the strength of the linear relationship between the two variables.
- Use correlation coefficients informally to describe the strength of the linear relationship illustrated by scatter plots.
- Distinguish between categorical and numerical variables.
- Use 2-way tables and analysis of cell frequencies and relative frequencies to help in deciding whether two categorical variables are related.
- Use standard deviation to measure variability in data distributions

8-1 Thinking with Mathematical Models: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Exploring Data Patterns	Investigation 2 Linear Models and Equations	Investigation 3 Inverse Variation	Investigation 4 Variability and Associations in Numerical Data	Investigation 5 Variability and Associations in Categorical Data
<p>Problem 1.1 Bridge Thickness and Strength FQ: How would you describe the relationship between bridge strength and bridge thickness revealed by your experiment?</p> <p>Problem 1.2 Bridge Length and Strength FQ: How would you describe the pattern relating bridge strength to bridge length shown in your experimental data?</p> <p>Problem 1.3 Custom Construction Parts: Finding Patterns FQ: How can you predict if a pattern between variables will be linear or nonlinear?</p>	<p>Problem 2.1 Modeling Linear Data Patterns FQ: How can you find a linear function that is a good model for a set of data and then measure the accuracy of that model with residuals?</p> <p>Problem 2.2 Up and Down the Staircase: Exploring Slope FQ: How do you write an equation for a linear function if you are given a graph, a table, or two points?</p> <p>Problem 2.3 Tree Top Fun: Equations for Linear Functions FQ: What strategies do you use in writing equations for linear functions?</p> <p>Problem 2.4 Boat Rental Business: Solving Linear Equations FQ: What strategies do you find useful to find solutions for linear equations?</p> <p>Problem 2.5 Amusement Park or Movies: Intersecting Linear Models FQ: When the graphs of two linear functions intersect, what do the coordinates of that intersection point tell you?</p>	<p>Problem 3.1 Rectangles with Fixed Area FQ: When the product of two variables is some fixed number, what is the pattern of change and how is that pattern of change reflected in tables and graphs of the relationship?</p> <p>Problem 3.2 Distance, Speed and Time FQ: What examples using distance, rate, and time show one variable inversely related to another?</p> <p>Problem 3.3 Planning a Field Trip: Finding Individual Cost FQ: How does the cost per person change if a fixed total cost is split among an increasing number of individual payers?</p> <p>Problem 3.4 Modeling Data Patterns FQ: What pattern in a table or graph of data suggests an inverse variation model and what strategies can you use to find an equation model for that kind of function?</p>	<p>Problem 4.1 Vitruvian Man: Relating Body Measurements FQ: If you have data relating two variables, how can you check to see whether a linear model is a good fit?</p> <p>Problem 4.2 Older and Faster: Negative Correlations FQ: From the scatter plot, how do you know if a linear model fits the data? How do you know if there are outliers? How do you know if the relationship is negative or positive?</p> <p>Problem 4.3 Correlation Coefficients and Outliers FQ: What does a correlation coefficient of 1, 0, or -1 suggest to you about the relationship between two variables?</p> <p>Problem 4.4 Measuring Variability: Standard Deviation FQ: How do you calculate the standard deviation for a data distribution and what does that statistic tell about the distribution?</p>	<p>Problem 5.1 Wood or Steel? That's the Question FQ: What does a two-way table show you about preferences among groups?</p> <p>Problem 5.2 Politics of Girls and Boys: Analyzing Data in Two-Way Tables FQ: Suppose you have recorded the counts of different preferences by group in a two-way table. How can you use those counts, or percents from the counts, to decide if two groups have the same preferences or not?</p> <p>Problem 5.3 After-School Jobs and Homework: Working Backward: Setting up a Two-Way Table FQ: Suppose you have data about the same trait in two groups. How can you organize the data to compare and decide if the groups are the same or not relative to the trait?</p>
<p>Mathematical Reflections</p> <p>1. You can represent a relationship between variables with a table, a graph, a description in words, or an equation. 1a. How can you decide whether a relationship is linear by studying the pattern in a data table? 1b. How can you decide whether a relationship is linear by studying the pattern in a graph? 1c. How can you decide whether a relationship is linear by studying the words used to describe the variables? 1d. How can you decide whether a relationship is linear by studying the equation that expresses the relationship in symbolic form?</p> <p>2. What are the advantages and disadvantages in finding patterns and making predictions?</p>	<p>Mathematical Reflections</p> <p>1. Why is it helpful to use a linear model for a set of data?</p> <p>2. When does it make sense to choose a linear function to model a set of data?</p> <p>3. How would you find the equation for a linear function in the following situations?</p> <p>3a. You are given a description of the variables in words. 3b. You are given a table of values for the variables 3c. You are given a graph of sample data points</p> <p>4. What strategies can you use to solve a linear equation such as $500 = 245 + 5x$?</p> <p>5. What kind of mathematical sentences express "at least" and "at most" questions about linear functions?</p>	<p>Mathematical Reflections</p> <p>1. Suppose the relationship between variables x and y is an inverse variation.</p> <p>1a. How do the values of y change as the values of x increase?</p> <p>1b. Describe the trend in a graph of (x, y) values.</p> <p>1c. Describe the equation that relates the values of x and y.</p> <p>2. How is an inverse variation similar to a linear relationship? How is it different?</p>	<p>Mathematical Reflections</p> <p>1. Think about the pattern of points you see in a scatter plot. 1a. What pattern would you expect when two variables are related by a linear model with positive slope? 1b. What pattern would you expect when two variables are related by a linear model with negative slope? 1c. What would you expect to see in a scatter plot when two variables are unrelated?</p> <p>2. You assessed the accuracy of linear models. 2a. What do outliers on a scatter plot indicate? 2b. What can you learn from the errors of prediction or residuals? 2c. What do you know about a linear model from the correlation coefficient?</p> <p>3. What does the standard deviation tell you about a set of data?</p>	<p>Mathematical Reflections</p> <p>1. What are categorical variables and what do they measure?</p> <p>2. Suppose a survey asked teenagers and adults whether or not they use text messaging. 2a. How could you arrange the data to compare the groups? 2b. How would you decide that the two groups - teenagers and adults - were different in their use of text messaging? 2c. Suppose that one analysis compared only the numbers in each group - teenage text messenger, teenage non-text messenger, adult text messenger, and adult non-text messenger. How might the analysis result in misleading conclusions?</p>

8-2 Looking for Pythagoras

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Pythagorean Theorem Understand and apply the Pythagorean Theorem

- Develop strategies for finding the distance between two points on a coordinate grid
- Explain a proof of the Pythagorean Theorem
- Use the Pythagorean Theorem and its converse to solve a variety of problems.
- Use the Pythagorean Theorem to find the equation of a circle with its center located at the origin

Real Numbers Understand the set of real numbers consists of rational and irrational numbers

- Interpret square roots and cube roots of numbers by making use of their related geometric representations
- Relate the area of a square to the side length of the square
- Estimate the values of square roots
- Estimate the values of cube roots
- Relate the volume of a cube to the edge length of the cube
- Compare numbers that can be represented as fractions (rational numbers) to numbers that cannot be represented as fractions (irrational numbers) and recognize that the set of real numbers consists of rational and irrational numbers.
- Represent rational numbers as fractions and as terminating decimals or repeating decimals
- Recognize that irrational numbers cannot be represented as fractions and are nonterminating, nonrepeating decimals
- Recognize that the square root of a whole number that is not a square is irrational
- Locate irrational numbers on a number line
- Use and understand properties of rational and irrational numbers.

8-2 Looking for Pythagoras: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Coordinate Grids	Investigation 2 Squaring Off	Investigation 3 They Pythagorean Theorem	Investigation 4 Using the Pythagorean Theorem: Understanding Rational numbers	Investigation 5 Using the Pythagorean Theorem: Analyzing Triangles and Circles
<p>Problem 1.1 Driving Around Euclid: Locating Points and Finding Distances FQ: How do driving distance between two coordinates relate to each other?</p> <p>Problem 1.2 Planning Parks: Shapes on a Coordinate Grid FQ: How do the coordinates of endpoints of a segment help draw other lines, which re parallel or perpendicular to the segment?</p> <p>Problem 1.3 Finding Areas FQ: How does knowing how to calculate areas of rectangles and triangles help n the calculation of irregular areas?</p>	<p>Problem 2.1 Looking for Squares FQ: How many different square areas are possible to draw using the dot grid as vertices? Why are some square areas not possible?</p> <p>Problem 2.2 Square Roots FQ: What does \sqrt{x} mean? How does it relate to x?</p> <p>Problem 2.3 Using Squares to Find Lengths FQ: How can you find the distance between any two points on a grid?</p> <p>Problem 2.4 Cube Roots FQ: What does it mean to take the cube root of a number?</p>	<p>Problem 3.1 Discovering the Pythagorean Theorem FQ: You know the sum of the two shortest side lengths of a triangle must be greater than the third side length. Is there a similar relationship among the squares on the sides of a triangle? Is the relationship the same for all triangles?</p> <p>Problem 3.2 A Proof of the Pythagorean Theorem FQ: How can you prove that the relationship observed in Problem 3.1 will work for all right triangles?</p> <p>Problem 3.3 Finding Distances FQ: How can you find the distance between any two points on a plane?</p> <p>Problem 3.4 Measuring the Egyptian Way: Lengths That Form a Right Triangle FQ: If a triangle with side lengths a, b, and c satisfies the relationship $a^2 + b^2 = c^2$, is the triangle a right triangle?</p>	<p>Problem 4.1 Analyzing the Wheel of Theodorus: Square Roots on a Number Line FQ: Dan you find distances that are exact square roots of all whole numbers? Can you order square roots on a number line?</p> <p>Problem 4.2 Representing Fractions as Decimals FQ: Why can you represent every fraction as a repeating or terminating decimal? How can you predict which representations will repeat and which will terminate?</p> <p>Problem 4.3 Representing Decimals as Fractions FQ: Can you represent every repeating or terminating decimal as a fraction?</p> <p>Problem 4.4 Getting Real: Irrational Numbers FQ: Can you identify every number as either rational or irrational?</p>	<p>Problem 5.1 Stopping Sneaky Sally: Finding Side Lengths FQ: How can you use the Pythagorean Theorem to find distances in a geometric shape?</p> <p>Problem 5.2 Analyzing Triangles FQ: How do the lengths of the sides of a 30-60-90 triangle relate to each other?</p> <p>Problem 5.3 Analyzing Circles FQ: What is the relationship between the coordinates of a point (x, y) on a circle with a center at the origin?</p>
<p>Mathematical Reflections</p> <ol style="list-style-type: none"> In the city of Euclid, how does the driving distance form one place to another compare to the flying distance? Suppose you know the coordinates of two landmarks in Euclid. How can you find the distance between the landmarks? What are some strategies for finding areas of figures drawn on a grid? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Describe how you would find the length of a line segment connecting two dots on dot paper. Be sure to consider horizontal, vertical, and tilted segments. <ol style="list-style-type: none"> Explain what it means to find the square root of a number. Explain whether or not a number can have more than one square root. <ol style="list-style-type: none"> Explain what it means to find the cube root of a number. Explain whether or not a number can have more than one cube root. 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Suppose you are given the lengths of two sides of a right triangle. Describe how you can find the length of the third side. Suppose two points on a grid are not on the same horizontal or vertical line. Describe how you can use the Pythagorean Theorem to find the distance between the points without measuring. How can you determine whether a triangle is a right triangle if you know only the lengths of its sides? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Give three examples of fractions with decimal representations that terminate. Give three examples of fractions with decimal representations that repeat. Give three examples of irrational numbers, including one irrational number greater than 5. How can you determine whether you can write a given decimal as a fraction? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Give at least two examples of ways in which the Pythagorean Theorem can be useful. Describe the special properties of a 30-60-90 triangle. What information do you need to write the equation of a circle with the center at the origin?

8-3: Growing, Growing, Growing

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Exponential Functions Explore problem situations in which two or more variables have an exponential relationship to each other

- Identify situations that can be modeled with an exponential function
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation
- Represent an exponential function with a table, graph, or equation
- Make connections among the patterns of change in a table, graph, and equation of an exponential function
- Compare the growth/decay rate and growth/decay factor for an exponential function and recognize the role each plays in an exponential situation
- Identify the growth/decay factor and initial value in problem situations, tables, graphs, and equations that represent exponential functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function
- Determine the values of the independent and dependent variables from a table, graph, or equation of an exponential function
- Use an exponential equation to describe the graph and table of an exponential function
- Predict the y -intercept from an equation, graph, or table that represents an exponential function
- Interpret the information that the y -intercept of an exponential function represents
- Determine the effects of the growth (decay) factor and initial value for an exponential function on a graph of the function
- Solve problems about exponential growth and decay from a variety of different subject areas, including science and business, using an equation, table, or graph
- Observe that one exponential equation can model different contexts
- Compare exponential and linear functions

Equivalence Develop understanding of equivalent exponential expressions

- Write and interpret exponential expressions that represent the dependent variable in an exponential function
- Develop the rules for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation
- Write and interpret equivalent expressions using the rules for exponents and operations
- Solve problems that involve exponents, including scientific notation

8-3 Growing, Growing, Growing: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Exponential Growth	Investigation 2 Examining Growth Patterns	Investigation 3 Growth Factors and Growth Rates	Investigation 4 Exponential Decay	Investigation 5 Patterns with Exponents
<p>Problem 1.1 Making Ballots: Introducing Exponential Functions FQ: What are the variables in this situation and how are they related?</p> <p>Problem 1.2 Requesting a Reward: Representing Exponential Functions FQ: In what ways are the relationships represented in a chessboard and ballot-cutting situations similar? Different?</p> <p>Problem 1.3 Making a New Offer: Growth Factors FQ: How does the growth pattern for an exponential function show up in a table, graph, or equation that represents the function and how does it compare to the growth pattern in a linear function?</p>	<p>Problem 2.1 Killer Plant Strikes Lake Victoria: y-intercepts Other Than 1 FQ: What information do you need to write an equation that represents an exponential function?</p> <p>Problem 2.2 Growing Mold: Interpreting Equations for Exponential Functions FQ: How is the growth factor and initial population for an exponential function represented in an equation that represents the function?</p> <p>Problem 2.3 Studying Snake Populations: Interpreting Graphs of Exponential Functions FQ: How is the growth factor and initial population for an exponential function represented in a graph that represents the function?</p>	<p>Problem 3.1 Reproducing Rabbits: Fractional Growth Patterns FQ: How is the growth factor in this Problem similar to that in the previous Problems? How is it different?</p> <p>Problem 3.2 Investing for the Future: Growth Rates FQ: How are the growth factor and growth rate for an exponential function related? When might you use each in an exponential growth pattern?</p> <p>Problem 3.3 Making a Difference: Connecting Growth Rate and Growth Factor FQ: How does the initial population affect the growth patterns in an exponential function?</p>	<p>Problem 4.1 Making Smaller Ballots: Introducing Exponential Decay FQ: How does the pattern of change in this situation compare to growth patterns you have studied in previous Problems? How does the difference show up in a table, graph, and equation?</p> <p>Problem 4.2 Fighting Fleas: Representing Exponential Decay FQ: How can you recognize an exponential decay function from a contextual setting, table, graph, and equation that represents the function?</p> <p>Problem 4.3 Cooling Water: Modeling Exponential Decay FQ: How can you find the initial population and decay factor for an exponential decay relationship?</p>	<p>Problem 5.1 Looking for Patterns Among Exponents FQ: What patterns did you observe in the table of powers?</p> <p>Problem 5.2 Rules of Exponents FQ: What are several rules for working with exponents and why do they work?</p> <p>Problem 5.3 Extending the Rules of Exponents FQ: How are the rules for integral exponents related to rational exponents? How are the rules for exponents useful in writing equivalent expressions with exponents?</p> <p>Problem 5.4 Operations with Scientific Notation FQ: How does scientific notation help to solve problems?</p> <p>Problem 5.5 Revisiting Exponential Functions FQ: What are the effects of a and b on the graph of $y = a(b^x)$, $b \neq 0$.</p>
<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Describe an exponential growth pattern. Include key properties such as growth factors. How are exponential functions similar to and different from the linear functions you worked with in earlier Units? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> How can you use a table, a graph, and an equation that represent an exponential function to find the y-intercept and growth factor for the function? Explain. How can you use the y-intercept and growth factor to write an equation that represents an exponential function? Explain. How would you change your answers to Questions 1 and 2 for a linear function? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Suppose you know the initial value for a population and the yearly growth rate. <ol style="list-style-type: none"> How can you determine the population several years from now? How is a growth rate related to the growth factor for the population? How can you use this information to write an equation that models the situation? Suppose you know the initial value for a population and the yearly growth factor. <ol style="list-style-type: none"> How can you determine the population several years from now? How can you determine the yearly growth rate? Suppose you know the equation that represents the exponential function relating the population p and the number of years n. How can you determine the doubling time for the population? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> How can you recognize an exponential decay pattern from the following? <ol style="list-style-type: none"> a table of data a graph an equation How are exponential growth functions and exponential decay functions similar? How are they different? How are exponential decay functions and decreasing linear functions similar? How are they different? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> Describe some of the rules for operating with exponents. <ol style="list-style-type: none"> What is scientific notation? What are its practical applications? Describe the effects of a and b on the graph of $y = a(b^x)$. Compare exponential and linear functions. Include in your comparison information about their patterns of change, y-intercepts, whether the function is decreasing or increasing, and any other information you think is important. Include examples of how they are useful.

8-4: Frogs, Fleas and Painted Cubes

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Quadratic Functions Explore problem situations in which two variables are in a quadratic relationship

- Identify situations that can be modeled by quadratic functions
- Identify the pattern of change between two variables that represent a quadratic function in a situation, table, graph, or equation
- Determine values of the independent and dependent variables in a quadratic function from a table, graph, or equation
- Represent a quadratic function with a table, graph, and equation
- Make connections among the equation of a quadratic function, its graph, and the patterns of change in its table
- Use a quadratic equation to describe the characteristics of its graph and table
- Determine whether a quadratic function will have a maximum or a minimum point and predict the maximum or minimum point from its equation, graph, or table
- Predict the x - and y -intercepts from the equation, graph, or table of a quadratic function
- Predict the line of symmetry from an equation, graph, or table of a quadratic function
- Interpret the information that the x - and y -intercepts and maximum or minimum point represent
- Use an equation, graph, and table to solve problems involving quadratic relationships
- Observe that one quadratic equation can model different contexts
- Compare linear, quadratic, and exponential functions

Equivalence Develop an understanding of equivalent quadratic expressions

- Write and interpret a quadratic expression to represent the dependent variable in a quadratic function
- Use an area model to develop an understanding of the Distributive Property
- Use the Distributive Property to write equivalent quadratic expressions in expanded or factored form
- Select and interpret the appropriate equivalent quadratic expression (in factored or expanded form) for predicting the x - and y -intercepts, maximum or minimum point, and the line of symmetry for a graph of a quadratic function

8-4 Frogs, Fleas, and Painted Cubes: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Introduction to Quadratic Functions	Investigation 2 Quadratic Expressions	Investigation 3 Quadratic Patterns of Change	Investigation 4 Frogs Meet Fleas on a Cube: More Applications of Quadratic Functions
<p>Problem 1.1 Staking a Claim: Maximizing Area FQ: Describe the shape of a graph that represents the areas of rectangles with a fixed perimeter.</p> <p>Problem 1.2 Reading Graphs and Tables FQ: How does the maximum area of rectangles with a fixed perimeter appear in a graph or a table?</p> <p>Problem 1.3 Writing an Equation FQ: How can you write an equation for the areas of rectangles with a fixed perimeter?</p>	<p>Problem 2.1 Trading Land: Representing Areas of Rectangles FQ: If the length n of a square is increased by 2 units and its width n decreased by 2 units, what two equivalent expressions represent the area of the new figure?</p> <p>Problem 2.2 Changing Dimensions: The Distributive Property FQ: How does the Distributive Property apply to quadratic expressions? Explain.</p> <p>Problem 2.3 Factoring Quadratic Expressions FQ: What is a method for factoring an expression as a product of two or more factors? How is this related to the Distributive Property?</p> <p>Problem 2.4 Quadratic Functions and Their Graphs FQ: How can you use a quadratic equation to predict the x- and y-intercepts, maximum/minimum points, and line of symmetry of its graph?</p>	<p>Problem 3.1 Exploring Triangular Numbers FQ: How many dots (or squares) are in the n^{th} triangular number?</p> <p>Problem 3.2 Counting Handshakes: Another Quadratic Function FQ: If each team has n members, how many handshakes will occur?</p> <p>Problem 3.3 Examining Patterns of Change FQ: Describe the pattern of change between the number of people on a team and the number of handshakes that occur.</p> <p>Problem 3.4 Quadratic Functions and Patterns of Change FQ: Compare the pattern of change for a quadratic function to the patterns of change for linear and exponential functions.</p>	<p>Problem 4.1 Tracking a Ball: Interpreting a Table and an Equation FQ: How can you predict the maximum height of a ball from the graph of a quadratic function?</p> <p>Problem 4.2 Measuring Jumps: Comparing Quadratic Functions FQ: How can you predict the y-intercept of a quadratic function from its graph, table, or equation?</p> <p>Problem 4.3 Painted Cubes: Looking at Several Functions FQ: When a painted cube with edge length n is separated into n^3 small cubes, how many of these cubes will have paint on three faces? Two faces? One face? No faces?</p> <p>Problem 4.4 Putting It All Together: Comparing Functions FQ: What can you learn about a function from a table, graph, or equation that represents the function?</p>
<p>Mathematical Reflections</p> <p>1a. Describe the characteristics of graphs and tables of quadratic functions you have observed so far. 1b. How do the patterns in a graph of a quadratic function appear in the table of values for the function?</p> <p>2. Describe two ways to find the maximum area for rectangles with a fixed perimeter.</p> <p>3. How are tables, graphs, and equations for quadratic functions different from those for linear and exponential functions?</p>	<p>Mathematical Reflections</p> <p>1. Explain how you can use the Distributive Property to answer each question. Use examples to help with your explanations. 1a. Suppose a quadratic expression is in factored form. How can you find an equivalent expression in expanded form? 1b. Suppose a quadratic expression is in expanded form. How can you find an equivalent expression in factored form?</p> <p>2. Describe what you know about the shape of the graph of a quadratic function. Include important features of the graph and describe how you can predict these features from the equation of the function.</p>	<p>Mathematical Reflections</p> <p>1a. In what ways is the triangular-number relationship similar to the relationships in the handshake problems? In what ways are these relationships different? 1b. In what ways are the quadratic functions in this Investigation similar to the quadratic functions in Investigations 1 and 2? In what ways are they different?</p> <p>2a. In a table of values for a quadratic function, how can you use the pattern of change to predict the next value? 2b. How can you use a table of values to decide if a function is quadratic?</p> <p>3. Compare the patterns of change for linear, exponential, and quadratic functions.</p>	<p>Mathematical Reflections</p> <p>1. Describe three real-world situations that can be modeled by quadratic functions. For each situation, give examples of questions that quadratic representations help to answer.</p> <p>2. How can you recognize a quadratic function from 2a. a table? 2b. a graph? 2c. an equation?</p> <p>3. What clues in a problem situation indicate that a linear, exponential, or quadratic function is an appropriate model for the data in the problem?</p>

8-5: Butterflies, Pinwheels, and Wallpaper

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Transformations Describe types of transformations that relate points by the motions of Reflections, rotations, and translations, and describe methods for identifying and creating symmetric plane figures

- Recognize properties of Reflections, rotation, and translation transformations
- Explore techniques for using rigid motion transformations to create symmetric designs
- Use coordinate rules for basic rigid motion transformations

Congruence and Similarity Understand congruence and similarity and explore necessary and sufficient conditions for establishing congruent and similar shapes

- Recognize that two figures are congruent if one is derived from the other by a sequence of Reflections, rotation, and/or translation transformations
- Recognize that two figures are similar if one can be obtained from the other by a sequence of Reflections, rotations, translations, and/or dilations
- Use transformations to describe a sequence that exhibits the congruence between figures
- Use transformations to explore minimum measurement conditions for establishing congruence of triangles
- Use transformations to explore minimum measurement conditions for establishing similarity of triangles
- Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations
- Use properties of congruent and similar triangles to solve problems about shapes and measurements

8-5 Butterflies, Pinwheels and Wallpaper: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Symmetry and Transformations	Investigation 2 Transformations and Congruence	Investigation 3 Transforming Coordinates	Investigation 4 Dilations and Similar Figures
<p>Problem 1.1 Butterfly Symmetry: Line Reflections FQ: What does it mean to say that a figure has flip or reflectional symmetry? How is each point related to its image under transformation by reflection in a line?</p> <p>Problem 1.2 In a Spin: Rotations FQ: What does it mean to say that a figure has turn or rotation symmetry? How is each point related to its image under transformation by rotation?</p> <p>Problem 1.3 Sliding Around: Translations FQ: What does it mean to say that a figure has slide or translational symmetry? How is each point related to its image under transformation by translation?</p> <p>Problem 1.4 Properties of Transformations FQ: How, if at all, will the shape, size, and position of a figure change after each of the transformations – reflection, rotation, or translation?</p>	<p>Problem 2.1 Connecting Congruent Polygons FQ: What does it mean to say two geometric shapes are congruent to each other and how could you demonstrate congruence with movable copies of the figures?</p> <p>Problem 2.2 Supporting the World: Congruent Triangles I FQ: How much information do you need to decide that two triangles are probably congruent or not congruent? How do you go about planning transformations that ‘move’ one triangle onto another?</p> <p>Problem 2.3 Minimum Measurement: Congruent Triangles II FQ: What is the smallest number of side and/or angle measurements needed to conclude two triangles are congruent?</p>	<p>Problem 3.1 Flipping on a Grid: Coordinate Rules for Reflections FQ: How can you describe how points “move” under a reflection with coordinate rules in the form $(x,y) \rightarrow (\square, \square)$ when the reflection line is (1) the x-axis? (2) the y-axis? (3) the line $y=x$?</p> <p>Problem 3.2 Sliding on a Grid: Coordinate Rules for Translations FQ: What kind of coordinate rule $(x,y) \rightarrow (\square, \square)$ tells how to “move” any point to its image under a translation?</p> <p>Problem 3.3 Spinning on a Grid: Coordinate Rules for Rotations FQ: What are the coordinate rules that describe “motion” of points on a grid under turns of 90° and 180°?</p> <p>Problem 3.4 A Special Property of Translations and Half-Turns FQ: How are lines and their images under translations and half-turns related to each other?</p> <p>Problem 3.5 Parallel Lines, Transversals, and Angle Sums FQ: When two parallel lines are cut by a transversal, what can be said about the angles formed? What is always true about the angle measures in a triangle? How do you know that your answers are correct?</p>	<p>Problem 4.1 Focus on Dilations FQ: What coordinate rules model dilations and how do dilations change or preserve characteristics of the original figure?</p> <p>Problem 4.2 Return of Super Sleuth: Similarity Transformations FQ: How can you use transformations to check whether two figures are similar or not?</p> <p>Problem 4.3 Checking Similarity Without Transformations FQ: What information about the sides and angles of two triangles will guarantee that they are similar?</p> <p>Problem 4.4 Using Similar Triangles FQ: What facts about similar triangles allow you to find lengths in very large figures that you are unable to reach?</p>
<p>Mathematical Reflections</p> <ol style="list-style-type: none"> How would you explain to someone how to make a design with: <ol style="list-style-type: none"> reflectional symmetry? rotational symmetry? translational symmetry? How are points and their images related by each of these geometric transformations? <ol style="list-style-type: none"> reflections in line m rotation of d° about point P translation with distance and direction set by the segment from point X to point X'. How do reflections, rotations, and translations change the size and shape of line segments, angles, and/or polygons, if at all? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> How can you find a sequence of flips, turns, and slides to “move” one figure exactly onto another to show that they are congruent? What information about the sides and angles of two triangles will guarantee you can “move” one triangle onto the other? How could you convince someone that two given triangles are not congruent? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> What are the general forms of the coordinate rules for these transformations? <ol style="list-style-type: none"> reflection in the y-axis reflection in the x-axis counterclockwise rotation of 90° about the origin counterclockwise rotation of 180° about the origin translation that “moves” points a units horizontally and b units vertically What is the effect of translation and half-turns on lines? How has your knowledge of transformations changed or extended what you already knew about the angles formed by two parallel lines and a transversal? How has your knowledge of transformations changed or extended what you already knew about the sum of the angle measures of a triangle? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> How would you explain what it means for two geometric shapes to be similar using <ol style="list-style-type: none"> everyday words that most people could understand? technical terms of mathematics? Suppose you dilate a polygon to form a figure of a different size. How will the side lengths, angle measures, perimeters, areas, and slopes of the sides of the two figures be alike? How will they be different? <ol style="list-style-type: none"> How has your knowledge of dilations changed or extended what you already knew about similarity. What is the least amount of information you need in order to be sure that two triangles are similar? How do you use similarity to find the side lengths of similar figures?

8-6: Say It With Symbols

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Equivalence Develop understanding of equivalent expressions and equations

- Model situations with symbolic statements
- Recognize when two or more symbolic statements represent the same context
- Use the properties of real numbers, such as the Distributive Property, to write equivalent expressions
- Determine if different symbolic expressions are mathematically equivalent
- Interpret the information that equivalent expressions represent in a given context
- Determine the equivalent expression or equation that is most helpful in answering a particular question about a relationship
- Use algebraic equations to describe the relationship among the volumes of cylinders, cones and spheres that have the same height and radius
- Solve linear equations involving parentheses
- Determine if a linear equation has a finite number of solutions, an infinite number of solutions, or no solution
- Develop understanding and some fluency with factoring quadratic expressions
- Solve quadratic equations by factoring
- Recognize how and when to use symbols, rather than tables or graphs, to display relationships, generalizations, and proofs

Functions Develop an understanding of specific functions such as linear, exponential and quadratic functions

- Develop proficiency in identifying and representing relationships expressed in problem contexts with appropriate functions and use these relationships to solve the problem
- Analyze equations to determine the patterns of change in the tables and graphs that the equations represent
- Relate parts of a symbolic statement or expression to the underlying properties of the relationship they represent and to the context of the problem
- Determine characteristics of a graph (intercepts, maxima and minima, shape, etc.) of an equation by looking at its symbolic representation

8-6 Say It With Symbols: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Making Sense of Symbols: Equivalent Expressions	Investigation 2 Combining Expressions	Investigation 3 Solving Equations	Investigation 4 Looking Back at Functions	Investigation 5 Reasoning with Symbols
<p>Problem 1.1 Tiling Pools: Writing Equivalent Expressions FQ: What expression(s) represents the number of border tiles needed to surround a square pool with side length s?</p> <p>Problem 1.2 Thinking in Different Ways: Determining Equivalence FQ: How can you determine if two or more expressions are equivalent?</p> <p>Problem 1.3 The Community Pool Problem: Interpreting Expressions FQ: What information goes an expression represent in a given context?</p> <p>Problem 1.4 Diving In: Revisiting the Distributive Property FQ: What information does an expression represent in a given context?</p>	<p>Problem 2.1 Walking Together: Adding Expressions FQ: What are the advantages and disadvantages of using one equation rather than two or more equations to represent a situation?</p> <p>Problem 2.2 Predicting Profit: Substituting Expressions FQ: What are some ways that you can combine one or more expressions (or equations) to create a new expression (or equation)?</p> <p>Problem 2.3 Making Candles: Volumes of Cylinders, Cones, and Spheres FQ: What equations represent the relationships among the volumes of cylinders, cones, and spheres?</p> <p>Problem 2.4 Selling Ice Cream: Solving Volume Problems FQ: What formulas are useful in solving problems involving volumes of cylinders, cones, and spheres?</p>	<p>Problem 3.1 Selling Greeting Cards: Solving Linear Equations FQ: What strategies can you use to solve equations that contain parentheses?</p> <p>Problem 3.2 Comparing Costs: Solving More Linear Equations FQ: What are strategies for finding a solution that is common to two-variable linear equations?</p> <p>Problem 3.3 Factoring Quadratic Equations FQ: What are some strategies for factoring a quadratic expression?</p> <p>Problem 3.4 Solving Quadratic Equations FQ: What are some strategies for solving quadratic equations?</p>	<p>Problem 4.1 Pumping Water: Looking at Patterns of Change FQ: How can you use an equation to answer particular questions about a function and the situation it represents?</p> <p>Problem 4.2 Area and Profit – What’s the Connection? Using Equations FQ: How can two different contexts be represented by the same equation?</p> <p>Problem 4.3 Generating Patterns: Linear, Exponential, Quadratic FQ: How can you determine the patterns of change of a function from a table of data for the function?</p> <p>Problem 4.4 What’s the Function? Modeling With Functions FQ: How can you determine which function to use to solve or represent a problem?</p>	<p>Problem 5.1 Using Algebra to Solve a Puzzle FQ: How can you determine to use to solve or represent a problem?</p> <p>Problem 5.2 Odd and Even Revisited FQ: How can you use algebra to represent and prove a conjecture about numbers?</p> <p>Problem 5.3 Squaring Odd Numbers FQ: What are some strategies for making and proving a conjecture?</p>
<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. What does it mean to say that two expressions are equivalent? 2. Explain how you can use the Distributive Property to write equivalent expressions. 3. Explain how you can use the Distributive and Commutative properties to show that two or more expressions are equivalent. 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. Describe a situation in which it is helpful to add expressions to form a new expression. Explain how you can combine the expressions. 2. Describe a situation in which it is helpful to substitute an equivalent expression for a quantity in an equation. 3. What are the advantages and disadvantages of working with one equation rather than two or more equations in a given situation? 4. Write an expression that represents the volume of each three-dimensional figure. Explain your reasoning. <ol style="list-style-type: none"> 4a. cylinder 4b. cone 4c. sphere 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1a. Describe some general strategies for solving linear equations, including those with parentheses. Give examples that illustrate your strategies. 1b. Describe how you can tell if a linear equation has a finite number of solutions, an infinite number of solutions, or no solutions. 2. Describe some strategies for solving quadratic equations of the form $ax^2 + bx + c = 0$. Give examples. 3. How are the solutions of linear and quadratic equations related to graphs of the equations? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. Describe how you can tell whether an equation is a linear, an exponential, or a quadratic function. 2. Describe how you can determine specific features of the graph of a function from its equation. Include its shape, x- and y-intercepts, maximum and minimum points, and patterns of change. 3. Describe how you can recognize which function to use to solve an applied problem. 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. Describe how and why you could use symbolic statements to represent relationships and conjectures. 2. Describe how you can show that your conjectures are correct.

8-7: It's In The System

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear Equations Develop understanding of linear equations and systems of linear equations

- Recognize linear equations in two variables in standard form $Ax+By=C$
- Recognize that a linear equation in the form $Ax+By=C$ has infinitely many solutions (x,y) and the graph of those solutions is always a straight line
- Recognize that the form $Ax+By=C$ of linear equations is equivalent to the form $y=mx+b$ for linear equations
- Continue to develop skills in solving a linear equation in two variables by graphing and with algebraic methods
- Recognize that solving a system of linear equations is equivalent to finding values of the variables that will simultaneously satisfy all equations in the system
- Develop skills in solving systems of linear equations by graphing solutions of separate equations; writing the system of equations in equivalent $y=mx+b$ form; or using combinations of the system to eliminate one variable
- Recognize that systems of linear equations in the form $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$ may have exactly one solution, which is the intersection point of the lines represented by the equations; infinitely many solutions, which is represented by a single line for both equations; or no solution, which is represented by two parallel lines
- Choose between graphing and symbolic methods to efficiently find the solution to a particular system of linear equations
- Gain fluency with symbol manipulation in solving systems of linear equations
- Solve problems that involve systems of linear equations

Linear Inequalities Develop understanding of graphing and symbolic methods for solving linear inequalities with one and two variables

- Recognize differences between strict and inclusive inequalities
- Continue to develop skill in solving a linear inequality in two variables by graphing and symbolic methods
- Develop skill in solving systems of linear inequalities by graphing solutions of each inequality and finding the region of feasible points that satisfy both inequalities; and solving inequalities to find pairs of numbers that satisfy both inequalities
- Choose between graphing and symbolic methods to efficiently find the region of feasible points to a particular system of linear inequalities
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables symbolically and graphically
- Solve problems that involve linear inequalities or systems of linear inequalities

8-7 It's in the System: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Linear Equations With Two Variables	Investigation 2 Solving Linear Systems Symbolically	Investigation 3 Systems of Functions and Inequalities	Investigation 4 Systems of Linear Inequalities
<p>Problem 1.1 Shirts and Caps: Solving Equations With Two Variables FQ: What kind of solution will be found for an equation with two variables such as $3x + 5y = 13$? What will the graphs of those two solutions look like?</p> <p>Problem 1.2 Connecting $Ax + By = C$ and $y = mx + b$ FQ: How can change an equation from $Ax + By = C$ form to an equivalent $y = mx + b$ form and vice versa?</p> <p>Problem 1.3 Booster Club Members: Intersecting Lines FQ: What does it mean to find the common solution to two linear equations with two variables?</p>	<p>Problem 2.1 Shirts and Caps Again: Solving Systems With $y = mx + b$ FQ: How can you solve a system of two linear equations with two variables by writing each equation in equivalent $y = mx + b$ form? What are the solution possibilities for the system? How are solutions in the graph of the system?</p> <p>Problem 2.2 Taco Truck Lunch: Solving System by Combining Equations I FQ: How can you solve a system of linear equations by combining the two equations into a single equation using addition or subtraction?</p> <p>Problem 2.3 Solving Systems by Combining Equations II FQ: How can you rewrite the equations in a system to make an equivalent system or equation?</p>	<p>Problem 3.1 Comparing Security Services: Linear Inequalities FQ: How can you use graphs to find the solutions of an inequality such as $ax + b < cx + d$? How can the solutions be represented on a number-line graph?</p> <p>Problem 3.2 Solving Linear Inequalities Symbolically FQ: How does applying the same operation to each side of an inequality change (or not) the relationship of the two quantities being compared? How can linear inequalities be solved using strategies to strategies for solving linear equations?</p> <p>Problem 3.3 Operating at a Profit: Systems of Lines and Curves FQ: What are the possible solutions for a system that includes one linear and one quadratic function and how can you find these solutions?</p>	<p>Problem 4.1 Limited Driving Miles: Inequalities With Two Variables FQ: If a problem involves solving an inequality such as $ax + by \leq c$, how many solutions would you expect to find? What would a coordinate graph of the solution look like?</p> <p>Problem 4.2 What Makes a Car Green: Solving Inequalities by Graphing I FQ: What would a graph of solutions (in the first quadrant) to an inequality $Ax + By \leq C$ look like?</p> <p>Problem 4.3 Feasible Points: Solving Inequalities by Graphing II FQ: How can you predict the shape of the graph of an inequality from its symbolic statement ($Ax + By \leq C$)?</p> <p>Problem 4.4 Miles of Emissions: Systems of Linear Inequalities FQ: What are some strategies for solving a system of linear inequalities?</p>
<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. What pattern will result from plotting all points (x,y) that satisfy an equation in the form $Ax + By = C$? 2. How can you change linear equations in the form $Ax + By = C$ to $y = mx + b$ form and vice versa? Explain when one form might be more useful than the other. 3. How can you use a graph to find values of x and y that satisfy systems of two linear equations in the form $Ax + By = C$? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. What is the goal in solving a system of linear equations? 2. What strategies can you use to solve a system of linear equations? 3. How can you check a possible solution of a system of linear equations? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. How can you use coordinate graphs to solve linear equations such as $ax + b = cx + d$ and linear inequalities such as $ax + b < cx + d$? 2. How can you use symbolic reasoning to solve inequalities such as $ax + b < cx + d$? 3. What strategies can you use to solve systems of equations and inequalities that involve linear and quadratic functions or lines and circles? 	<p>Mathematical Reflections</p> <ol style="list-style-type: none"> 1. Suppose you are given one linear inequality with two variables. How could you use a graph to find solutions of the inequality? 2. Suppose you were given a system of two linear inequalities. How could you use a graph to find solutions of the system?

8-8: Function Junction

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Functions Understand equivalence of algebraic expressions and functions

- Describe domain and range of functions
- Use $f(x)$ notation to describe and operate with functions
- Construct and interpret inverses of functions
- Analyze function rates of change using graphs
- Identify contexts and graphs of step and piecewise defined functions
- Analyze polynomial functions and their graphs
- Identify, analyze, and solve problems related to arithmetic and geometric sequences
- Compare arithmetic and geometric sequences to linear and exponential functions
- Recognize and solve problems using special kinds of functions

Equivalence Understand equivalence of algebraic expressions and functions

- Connect expressions for functions whose graphs are related by translation and/or stretching
- Develop and use vertex form to graph quadratic functions and solve quadratic equations
- Connect polynomial expressions and graphs of the polynomial functions they define, in order to identify max/min points, intercepts, and solutions of equations
- Use completing the square to write quadratics in equivalent vertex form
- Develop the quadratic formula for solving equations
- Develop complex numbers and operations
- Develop algorithms for adding, subtracting, and multiplying polynomials